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Presented is a fused course in plane, solid, and coordinate geometry. Elementary set theory, logic, and the principle of separation provide unifying threads throughout the text. The two curriculum guides have been prepared for use with two different texts. Either curriculum guide may be used depending upon the choice of the teacher and the needs of the students.

Guide A is presented in eleven units with one introductory unit on sets. Units are as follows: points, lines, planes, and space; triangles; essentials in proof; introduction to formal proof; parallel and perpendicular lines and planes; similarities; coordinate geometry; polygons and polyhedrons; circles and spheres; areas of plane figures; and areas and volumes of solid figures.

Guide B is presented in thirteen units as follows: elements of geometry; induction; a method of discovery; deduction and proof; angle relationships and perpendicular lines; parallel lines and planes; congruent triangles; similar triangles; circles, arcs, and angles; methods of coordinate geometry; proofs using coordinate geometry; areas of polygons and circles; areas and volumes of solids; constructions and loci. Cross references are given in each guide for each of the two textbooks used.

The syllabus is based on much of the thinking of the commission on mathematics of the College Entrance Examination Board and the School Mathematics Study Group.

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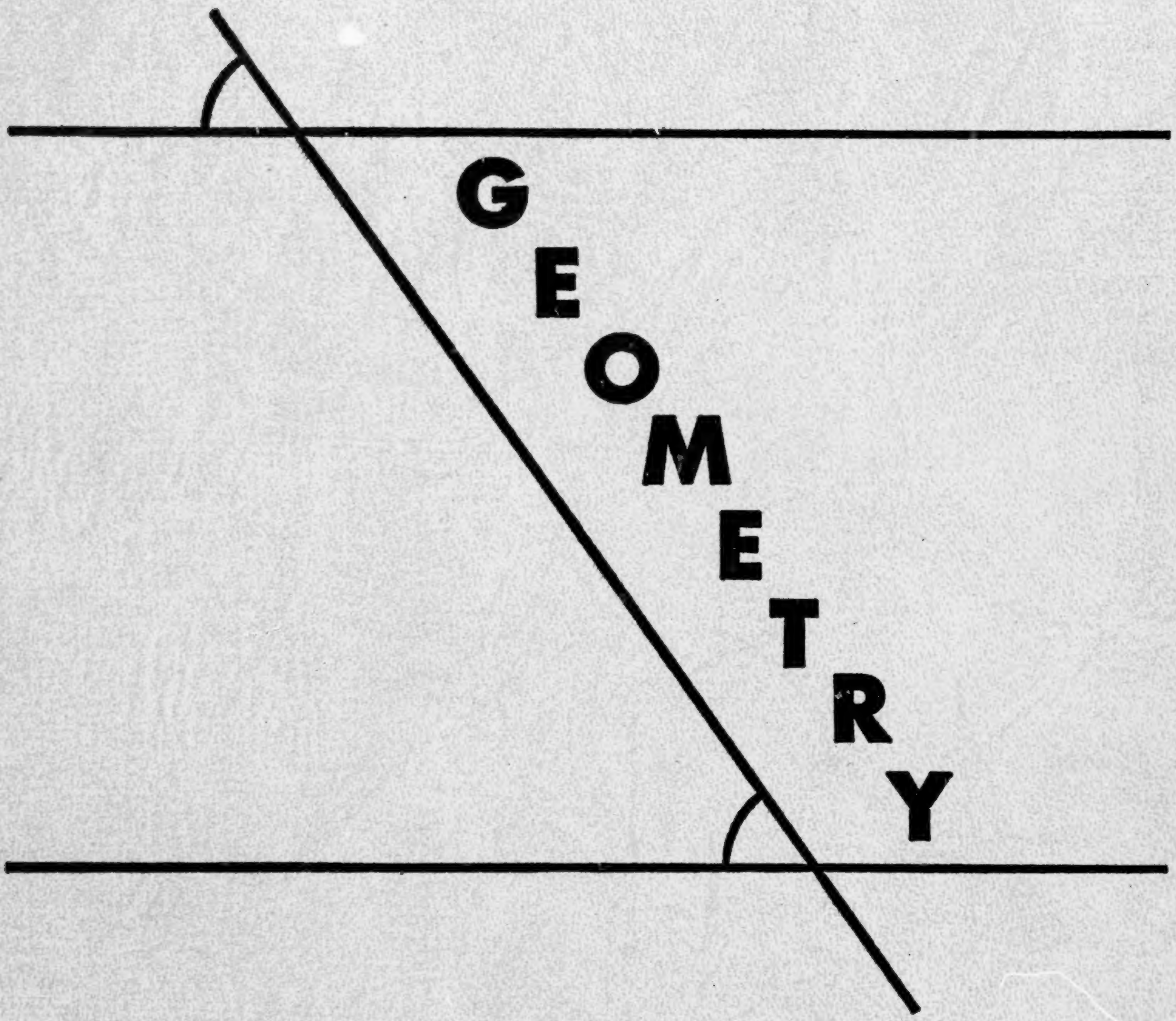
SENIOR HIGH SCHOOL
CURRICULUM GUIDE

GEOMETRY



BALTIMORE COUNTY PUBLIC SCHOOLS

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BALTIMORE COUNTY PUBLIC SCHOOLS

Tentative Guides

GEOMETRY

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Towson, Maryland
1963

FOREWORD

The fusion of geometry with algebra is characteristic of the development of mathematics, in which boundary lines between the divisions of the subject become blurred and eventually erased. Only recently has it been realized that fusion can profitably be begun on the high school level.

This syllabus presents a fused course in plane, solid, and coordinate geometry in recognition of and in agreement with the recommendations of the Commission on Mathematics of the College Entrance Examination Board and the School Mathematics Study Group. Elementary set theory, logic, and the principle of separation provide unifying threads throughout this approach to geometry.

The course of study, Geometry, actually contains two different curriculum guides for an updated approach to geometry. Curriculum Guide A and Curriculum Guide B have been prepared for use with two different texts. The decision of the committee to write two different outlines was based on the fact that both texts present a modern approach in different sequences. It is consistent with county policy to offer a choice of texts whenever possible. The selection and proper use of either of these guides should enable the teacher to present a modern course in geometry suited to the needs of Baltimore County students.

The guides are tentative and presume continuous teacher evaluation. Teachers will use the materials, modifying them to fit the mathematical backgrounds and capabilities of the class. Teacher modifications may be incorporated into a revision of the program at a later date.

The Board of Education and the Superintendent of Schools wish to express their appreciation to the curriculum committee and to all the mathematics teachers of Baltimore County whose efforts made possible the development of this curriculum guide.

Special commendation is due Mrs. Margaret Einstein, secretary to the committee, for her careful and painstaking efforts in the production of this syllabus.

William S. Sartorius
Superintendent of Schools

Towson, Maryland
September 1963

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CURRICULUM GUIDE A

GEOMETRY

A Tentative Guide

Prepared for Use with the Text

Modern Geometry - Its Structure and Function

by

Henderson, Pingry, and Robinson

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BASIC CONSIDERATIONS AND SPECIFIC SUGGESTIONS

The purpose of this syllabus is to help you to understand and to teach geometry from a modern point of view. It should help you to improve your teaching by

assisting you in the interpretation of the basic text

serving as the core around which you can plan your daily lessons

providing you with teaching techniques which are the outgrowth of successful teaching experiences in classroom situations similar to your own.

Since the basic text provides a wealth of explanation and a great variety of problems, the outline deviates from the text if and only if the deviation will result in a course which is more consistent with the philosophy of modern mathematics and more in keeping with the mathematical background and needs of the students in the Baltimore County Public Schools.

Modern geometry is distinguished from traditional geometry by two characteristics:

the fusion of plane, solid, and coordinate geometry into a single course

the use of a few basic concepts to unify and clarify subject matter.

These two characteristics provided the basis for planning the sequence of topics and the content of this course.

The Fusion of Plane, Solid, and Coordinate Geometry

The Selection of Theorems from Plane Geometry

This guide highlights the basic theorems of the text which should be emphasized as an integral part of a developmental chain of ideas. It also brings to the attention of the teacher certain problems in the basic text which may be regarded as theorems after they have been proved by the class. The inclusion of these secondary theorems in exercises gives the pupil meaningful practice in the proof of problems which he will find convenient to use in his development of geometry. The guide also indicates theorems which should be treated informally because they are not necessary to the further development of the subject.

The Role of Solid Geometry

Although the textbook presents much of the work in solid geometry in a very formal manner, the syllabus suggests that much of the solid geometry be taught informally as an extension of the topics from plane geometry. This practice is consistent with the philosophy expressed in the report of the Commission on Mathematics of the College Entrance Examination Board, and it is necessary if the entire course is to be completed in a regular school year. Teachers should help the students to understand that the solid geometry which they are learning is not a part of the deductive system but is inductive in nature. The use of the terms "basic vocabulary and generalizations" rather than "definitions and theorems" should help to emphasize this difference.

The Role of Coordinate Geometry

The inclusion of coordinate geometry adds flexibility to the course by providing the students with an additional method of proof. In many cases, proof by coordinate methods is more direct than the usual synthetic approach. Such flexibility was impossible in the day of Euclid because he lacked an adequate algebra for this purpose. With the development of algebra it became possible to postulate a one-to-one correspondence between points and numbers which can be readily applied to the geometry of a line, a plane, or space.

The basic text develops the concepts of coordinate geometry in a logical manner, but it fails to apply these concepts in proof. The outline lists theorems which should be proved by coordinate methods rather than by the usual synthetic method. For the convenience of the teacher, the proofs of these theorems are included in the appendix.

The Use of Basic Concepts

The Use of Sets

In the basic text the subject of sets is developed as it is needed to clarify geometric concepts. This approach would be ideal if the mathematics background of the pupils did not include sets. However, the topic of sets is rapidly becoming an integral part of the algebra courses in Baltimore County schools. For this reason, it is suggested that the teacher review briefly the vocabulary relating to sets and begin to use it immediately in the presentation of topics.

Closely related to the concept of sets is the idea of separation. This concept is emphasized throughout the syllabus and provides a continuous thread in the development of geometry.

The Use of Logic

The basic text follows the same practice in the use of logic as it does in the use of sets. The syllabus departs from this practice by pulling together the various topics relating to logic and teaching them under the heading "Essentials of Proof." This is preferable because it helps the student relate and unify the basic principles of logic and thus obtain a better understanding of the nature of proof.

The Use of Limits

Although the study of limits in a rigorous fashion is a part of the study of calculus, the intuitive concept of limits can be applied to many topics in the study of geometry. The syllabus stresses the use of limits in the study of tangents and secants, circles and polygons, cylinders and prisms , cones and pyramids. The use of limits helps the student to organize the subject matter of geometry by thinking of the relationships that exist between two sets rather than considering each set as a separate entity.

The Use of Right Triangles

The Pythagorean Theorem is the most useful theorem in the study of geometry. The theorems which lead to its proof and the applied theorems which follow constitute a very basic chain in the development of a course in geometry. The use of right triangles and the relationships that exist in special right triangles are also essential to the development of many basic concepts in trigonometry. Special techniques for teaching this topic and a variety of exercises relating to the topic are included in the appendix.

Specific Suggestions

The Use of Student Notebooks

Each student should be required to keep a notebook which would include lists of important definitions, postulates, and theorems. These lists will be most useful to the student if he classifies them under headings relating to methods of proof. Some suggested headings are:

Methods of Proving Segments Equal
Methods of Proving Angles Equal
Methods of Proving Angles Supplementary
Methods of Proving Triangles Congruent
Methods of Proving Lines Perpendicular
Methods of Proving a Triangle is a Right Triangle
Methods of Proving a Triangle is Isosceles
Methods of Proving Lines are Parallel

The Use of Drills

A short drill at the beginning of the period should be a part of each lesson. It should be placed in a designated area so that students can begin work on it as soon as they come into the classroom. Each drill should be designed to serve at least one of three purposes:

> To lead to the intuitive development of a concept which is to be developed formally in the lesson
>
> To provide necessary practice of important concepts over an extended period of time. This often results in more effective learning than that achieved by overconcentration on a topic for a limited period of time
>
> To review basic concepts of algebra which are needed in the development of this approach to geometry. It is suggested that drills be used in teaching much of the material on real numbers in Unit Three. Teachers should include numerical examples of these properties in drills early in the year rather than wait for the topic to appear in the syllabus.

The Use of Time

The following chart should serve as a guide for the teacher to use in planning work. Only 175 days are scheduled. This should provide for adjustments which may be necessary due to school schedules and different ability levels of the various classes.

Unit	Name	Number of Days	Cumulative Total
	Introductory Unit - Sets	2	2
One	Points, Lines, Planes, and Space	12	14
Two	Triangles	6	20
Three	Essentials of Proof	12	32
Four	Introduction to Formal Proof	17	49
Five	Parallel and Perpendicular Lines and Planes	20	69
Six	Similarity	20	89
-------------------End of First Semester---------------------			
Seven	Coordinate Geometry	32	121
Eight	Polygons and Polyhedrons	20	141
Nine	Circles and Spheres	20	161
Ten	Areas of Plane Figures	8	169
Eleven	Areas and Volumes of Solid Figures	6	175

MATERIALS OF THE COURSE IN GEOMETRY

Modern Geometry: Its Structure and Function by Henderson, Pingry and Robinson will be used as the basic text.

The references to the right of the scope in the outline indicate helpful or additional materials for the teacher and pupils.

Code		Name of Reference
JDD	-	Jurgensen, Donnelly, and Dolciani. *Modern Geometry: Structure and Method*
TM	-	_______________ *Modern Geometry: Structure and Method* - "Teacher's Manual" (included in Teacher's Edition of the above text.)
HPR	-	Henderson, Pingry, and Robinson. *Modern Geometry: Its Structure and Function*
MZ	-	Morgan and Zartman. Geometry: *Plane* · *Solid* · *Coordinate*
SMSG	-	School Mathematics Study Group. *Mathematics for High School: Geometry with Coordinates*
C	-	Coffman. *Logic*
BK	-	Brant and Keedy. *Elementary Logic for Secondary Schools*
J	-	Johnson. *Logic and Reasoning in Mathematics*

SCOPE	REFERENCES	
	CODE	PAGES
INTRODUCTORY UNIT		
SETS		
(Time allotment - 1-2 days)		
This unit should be a review for the students. The time required for this unit will depend upon their background. Under no circumstances should the unit be expanded to a complete treatment of sets.		
I. Concept of set and Venn diagrams	HPR	30, 174
II. Element or member of a set ($\in$)	HPR	30-31
III. Domain of a variable (replacement set)	HPR	32
IV. Method of representing a set	HPR	34-36
A. Roster (listing) method		
B. Rule method		
V. Finite and infinite sets		
VI. Relationships between sets		
A. Identical sets (This text avoids the use of equal sets.)	HPR	36

SCOPE	REFERENCES	
	CODE	PAGES
B. Subsets	HPR...	37
1. Proper subset $(A \subset B)$		
2. Improper subset $(A = B)$		
3. The empty or null set (ϕ)	HPR...	172
C. Overlapping sets		
D. Disjoint sets		
VII. Operations on sets	HPR...	173
A. Intersection $(A \cap B)$		
B. Union $(A \cup B)$		
C. Complement $(\overline{A})$		

SCOPE	REFERENCES	
	CODE	PAGES

UNIT ONE

POINTS, LINES, PLANES, AND SPACE - SUBSETS OF SPACE

(Time allotment - 12 days)

This unit lays the foundation for a logical development of geometry. A number of basic concepts are developed. Some of these concepts will be unfamiliar to the student; for instance, the idea of separation. Other concepts will be a review of familiar ideas for the purpose of reinforcement. Although many definitions are formalized at this time, no attempt should be made to formalize any postulates. Basic concepts and definitions should be presented in a meaningful fashion. Mere parroting of a definition by the student does not imply understanding. Students should be encouraged to formulate their own definitions as long as they do not violate the essentials of a good definition.

Throughout this unit, as well as throughout all subsequent units, a number of parenthetical remarks have been inserted to:

1. Provide "hints" to the teacher
2. Indicate points to be emphasized
3. Indicate pitfalls to be avoided
4. Clarify the point of view taken in the basic text
5. Indicate alternate definitions to those found in the text

I. Role of undefined terms and definitions

A. Necessity and importance of undefined terms HPR...2

SCOPE	REFERENCES	
	CODE	PAGES
B. Definitions	MZ....	7-8
1. Necessity and importance	SMSG..	15-16
2. Characteristics of a good definition		
a. The term must be stated.		
b. The term must be expressed as a subset of a set.		
c. The definition must indicate how the term is different from all the other members of the set.		
d. The definition must not contain any unnecessary properties.		
e. The definition must be reversible.		
II. Basic Concepts and notation		
A. Undefined terms		
1. Point		
2. Line		
3. Plane		
B. Properties relating to undefined terms		
1. A line contains at least two points; a plane contains at least three points not all on one line.		
2. Two points determine a line.		
3. If two points of a line lie in a plane, then every point of the line lies in the plane.		

SCOPE	REFERENCES	
	CODE	PAGES
4. If two lines intersect, then their intersection is a single point. (The teacher should point out if the intersection is more than one point, at least two, then only one line is determined.)		
5. If two distinct planes have a point in common, then they have a second point in common, and thus a line in common.		
C. Intersection of two sets	HPR...	12
1. Two lines (The teacher should indicate to the student that if there are two intersecting lines, then there must be at least three noncollinear points. This would imply that two intersecting lines determine a plane.)		
2. A line and a plane		
3. Two planes		
D. Defined terms	HPR...	3
1. Space: the set of all points	MZ....	10
a. Space contains at least four points not all in one plane.		
b. Distinct points	SMSG..	36
c. Collinear and noncollinear points		
d. Coplanar and noncoplanar points		

SCOPE	REFERENCES	
	CODE	PAGES
2. Subsets of a line	MZ	10
a. Half-line	SMSG	135
(This term is not found in the text, but it is advisable to introduce it because it is basic to the concept of separation.)		
(1) Separation: A point separates a line into three distinct subsets.		
(a) The point		
(b) The two half-lines		
(Since a line has no measure, the term half-line cannot mean half of a line.)		
(2) Types of half-lines		
(a) Open half-line (point excluded)	MZ	10
(b) Closed half-line (point included)		
b. Ray: a closed half-line		
(1) Endpoint (origin) of a ray		
(2) Interior of a ray		
(3) Ray opposite a given ray		
(4) Notation		
(The teacher may wish to use the SMSG notation. $\overrightarrow{AB}$ denotes the ray AB.)		

SCOPE	REFERENCES	
	CODE	PAGES
c. Line segment	HPR	3-4
(1) Endpoints of a segment		
(2) Interior of a segment	MZ	11-13
(3) Notation		
(The teacher may wish to use the SMSG notation. $\overline{AB}$ denotes the segment AB.)		
(4) Measure of a segment		
(5) Midpoint of a segment		
(6) Bisector of a segment		
(A bisector of a segment can be a point, line, segment, ray, or plane.)		
3. Subset of a plane: half-plane	MZ	10
a. Separation: A line separates a plane into three distinct subsets.		
(1) The line (edge)	HPR	13
(2) The two half-planes		
b. Types of half-planes		
(1) Open half-plane (edge excluded)		
(2) Closed half-plane (edge included)		
(The half-plane defined in this text is a closed half-plane. This permits the use of a straight angle.)		

SCOPE	REFERENCES	
	CODE	PAGES
4. Subset of space: half-space		
a. Separation: A plane separates space into three distinct subsets.		
(1) The plane (2) The two half-spaces		
b. Types of half-spaces		
(1) Open half-spaces (plane excluded) (2) Closed half-space (plane included)		
5. Angles in a plane	HPR...	5
(An alternate definition is: An angle is the union of two rays with a common endpoint.)		15
a. Parts of an angle	SMSG..	143-145
(1) Vertex (2) Sides		
b. Notation		
(1) Three capital letters (2) A single letter: either capital or lower case (3) A numeral (4) Two case letters		
(The designation of an angle by two letters as used in this text is not in general use and should be avoided.)		

SCOPE	REFERENCES	
	CODE	PAGES

c. Separation: An angle separates a plane into three distinct subsets.

(1) If both sides of the angle do not lie in the edge of the half-plane, the subsets consist of:

(a) The points in the angle
(b) The points in the interior of the angle
(c) The points in the exterior of the angle

(The interior and exterior of an angle should be defined.)

(2) If both sides of the angle lie in the edge of the half-plane, the subsets consist of:

(a) The points in the angle
(b) The points in the two open half-planes

(A straight angle has no interior.)

d. Measure of an angle

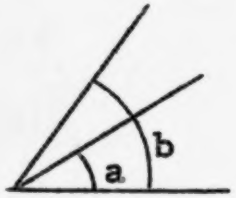
(1) Equal angles

(Many modern texts use "congruent angles" and "congruent segments" for those sets which have the same measure. These same texts state that two sets are equal if and only if they have the same members. In other words, when two angles are equal, we mean that there are two different names for the same set. This text, however, uses the

SCOPE	REFERENCES	
	CODE	PAGES
terms "equal angle " and "equal segments" for those sets which have the same measure. For those sets whose members are the same, the term "identical sets" should be used.)		
(2) Midray of an angle		
(A midray divides an angle into two equal angles.)		
(3) Bisector of an angle		
(The bisector of an angle may be a ray, line, or plane.)		
(4) Angles classified by measure	HPR...	17
(a) Straight angle	MZ....	24
(A straight angle is an angle whose interior is empty Since zero angles and angles with measures greater than 180 are not necessary for the logical development of geometry, these angles are not included in the text.)		
(b) Right angle (c) Acute angle (d) Obtuse angle (e) Oblique angle		

SCOPE	REFERENCES CODE | PAGES
6. Pairs of lines and angles	HPR...22-23
a. Overlapping angles (It is the interiors of the angles which overlap.)	
b. Adjacent angles (It is suggested that the teacher use the following illustrations to clarify the idea of adjacent and non-adjacent angles.)	SMSG..181 MZ....24-32

(1)



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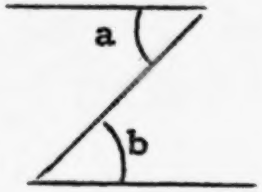

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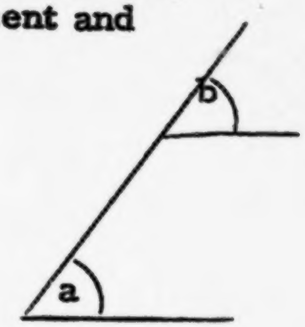

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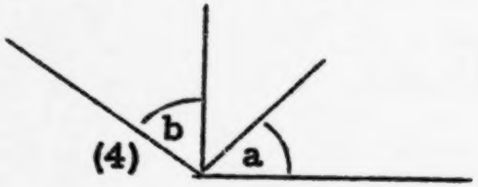

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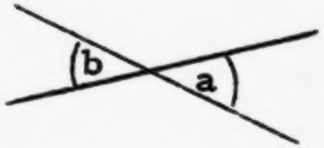

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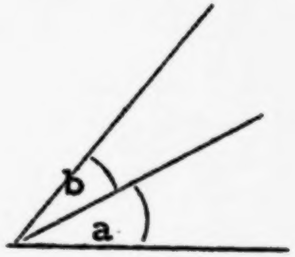


(Students should be questioned as to why figures (1) - (5) are non-adjacent angles.)

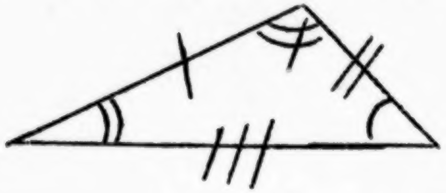
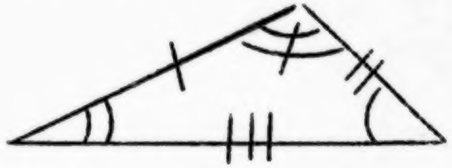
SCOPE	REFERENCES	
	CODE	PAGES
c. Complementary angles		
d. Supplementary angles		
(The teacher should stress that complementary and supplementary angles are two angles which are not necessarily adjacent.)	JDD	40-41
e. Vertical angles	SMSG	179
f. Linear pair: two angles which are formed by three rays with a common endpoint, two of which are opposite rays		
g. Perpendicular lines		
h. Perpendicular bisector of a line segment	HPR	169
i. Skew lines		
7. Dihedral angles		
a. Parts of a dihedral angle		
(1) Face (2) Edge		
(3) Plane angle of a dihedral angle	HPR	26
b. Measure of a dihedral angle		
(A dihedral angle has the same measure as its plane angle.)		

SCOPE	REFERENCES CODE	REFERENCES PAGES
c. Types of dihedral angles and pairs of dihedral angles (Dihedral angles and pairs of dihedral angles are classified according to their plane angles.)		
E. Methods of determining lines and planes	MZ. . . .	9
1. Two points determine a line. 2. Three noncollinear points determine a plane. 3. A line and a point not on the line determine a plane. (The teacher should recognize that statements 1 and 2 are postulates, while statement 3 is a theorem. Statement 3 enables the teacher to provide the student with an informal treatment of logical proof. Students should be led to recognize statement 3 as a logical consequence of statements 1 and 2.)	SMSG. .	33-48
F. Constructions (In presenting this section, the teacher can use the overhead projector and overlays to good advantage.)	HPR. . .	8 19 20 28
1. A line segment equal to a given segment 2. An angle equal to a given angle 3. The perpendicular bisector of a segment 4. The bisector of an angle 5. A perpendicular to a line through a point on the line (The teacher may wish to treat this as a bisector of a straight angle.) 6. A perpendicular to a line through a point not in the line		

SCOPE	REFERENCES CODE | PAGES
UNIT TWO	
TRIANGLES	
(Time allotment - 7 days)	
This unit presents the basic vocabulary needed in a discussion of triangles and informally introduces congruence of triangles. The filmstrip Properties of Triangles, by McGraw-Hill, develops the subject matter of the unit in an interesting manner. It is suggested that frames 1-27 be used in teaching Section I and frames 28-39 be used in the development of the concepts in Part D of Section II.	
I. Triangle	
A. Definition	
B. Notation	**HPR...7**
C. Separation: A triangle separates a plane into three distinct subsets.	**SMSG..201-202**
1. The points in the triangle	
2. The points in the interior of the triangle	
(The teacher should use the overhead projector to show that the interior of a triangle is the intersection of the interiors of the angles of the triangle.)	
3. The points in the exterior of the triangle	

SCOPE	REFERENCES	
	CODE	PAGES
D. Parts of a triangle		
1. Vertices		
2. Sides	HPR	7
3. Angles		
E. Nomenclature		
(The relationships of triangles should be illustrated by the use of set diagrams.)		
1. Triangles classified by sides		
a. Scalene: a triangle with no two sides equal		
b. Isosceles: a triangle with at least two sides equal		
(1) Legs		
(2) Base		
(3) Vertex angle		
(4) Base angles		
c. Equilateral: a triangle with three equal sides		
2. Triangles classified by angles		
a. Right: a triangle containing one right angle		
(1) Right angle		
(2) Hypotenuse		
(3) Legs		

SCOPE	REFERENCES	
	CODE	PAGES
b. Obtuse: a triangle containing one obtuse angle		
c. Acute: a triangle containing three acute angles		
F. Important lines in triangles		
1. Altitude	HPR	403
2. Median	HPR	403
3. Angle bisector	HPR	295
II. Correspondence and congruence		
A. Physical concept of congruence	SMSG	225
1. Same size and same shape		
2. Coincidence		
B. Corresponding parts of two triangles	HPR	48
1. Three pairs of corresponding sides	SMSG	227-228
2. Three pairs of corresponding angles		
C. Definition of congruence		
D. Methods of proving triangles congruent		
1. Experiments with triangles	HPR	46-50
2. Properties of congruence		
a. SAS		
b. ASA		
c. SSS		

SCOPE	REFERENCES	
	CODE	PAGES
E. Identification of corresponding parts of congruent triangles 1. Included side 2. Included angle 3. Side opposite given angle 4. Angle opposite given side (The teacher should encourage the student to identify corresponding parts with special markings.)  	HPR... SMSG..	47; 53-54 253-255
F. Use of symbolism 1. $\triangle ABC \cong \triangle XYZ$ 2. $\angle A \longleftrightarrow \angle X$ $AB \longleftrightarrow XY$ $\angle B \longleftrightarrow \angle Y$ $BC \longleftrightarrow YZ$ $\angle G \longleftrightarrow \angle Z$ $AC \longleftrightarrow XZ$	HPR... SMSG.,	56-58 226-230
G. Informal solution of problems (The teacher will find the use of the overhead projector helpful in the discussion of overlapping triangles.)	HPR...	59-63

SCOPE	REFERENCES	
	CODE	PAGES

UNIT THREE

ESSENTIALS OF PROOF

(Time allotment - 12 days)

Although this unit cover the material in the basic text from pages 64-133, the organization of the topics in the outline does not follow the organization of the topics in the text. The reasons for this departure are:

1. The material in the text is not organized around central ideas, and it is difficult for students to shift from one topic to another and gain an adequate understanding of the concepts presented.

2. Some terms other than undefined terms are used in definitions before they are defined.

3. The presentation of properties of real numbers is too rigorous for many students. The proofs of the properties of real numbers are included in the Algebra III-IV course, and they should be omitted at this time. However, these properties should be made meaningful to the students through the use of numerical examples.

It is very important that teachers do not become "bogged down" with the rigor of the material in this section. After presenting and discussing numerical examples of the properties of real numbers with the class, teachers should include problems of a similar nature in daily drills. It is preferable to review these properties in drills over an extended period of time rather than to work for complete mastery of them in a few days.

SCOPE	REFERENCES CODE	REFERENCES PAGES
Teachers who wish to use truth tables in the section on logic will find the following references helpful:		
Kemeny, Snell, Thompson. Introduction to Finite Mathematics. pp. 1-53 Johnson. Logic and Reasoning in Mathematics. Brant and Keedy. Elementary Logic for Secondary Schools. Coffman. Logic. (mimeographed unit)		
I. Sentence forms in logic		
A. Basic concepts		
1. A statement is a declarative sentence which can be judged true or false but not both.	HPR...	64-65
2. A statement is represented by a lower case letter such as p or q.		
3. The negative of a statement is called a negation; it is represented by the symbol $\sim p$ and is read "not p."		
4. The truth table for the negation of a statement is as follows:		

p	$\sim p$
T	F
F	T

SCOPE

REFERENCES
CODE | PAGES

B. Conditional statements

1. Definition of a conditional statement HPR...66-67

2. Form of a conditional statement

   a. If p, then q
   b. p implies q: $p \longrightarrow q$

3. Parts of a conditional statement

   a. Antecendent
   b. Consequent

4. Truth value of a conditional statement HPR...67

   a. Definition of truth value for a conditional statement

   b. Truth table for a conditional statement (optional)

p	q	$p \longrightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

(The last two rows of the truth table are not needed in the development of geometry. We are interested only in the cases where the antecedent of the statement is true.)

SCOPE	REFERENCES CODE | PAGES
5. Variants of the conditional and their truth tables	
a. Name of variant	HPR...192-194

(1) Converse: $q \longrightarrow p$
(2) Inverse: $\sim p \longrightarrow \sim q$
(3) Contrapositive: $\sim q \longrightarrow \sim p$

b. Truth tables (optional)

				Conditional	Converse	Inverse	Contrapositive
p	q	$\sim p$	$\sim q$	$p \longrightarrow q$	$q \longrightarrow p$	$\sim p \longrightarrow \sim q$	$\sim q \longrightarrow \sim p$
T	T	F	F	T	T	T	T
T	F	F	T	F	T	T	F
F	T	T	F	T	F	F	T
F	F	T	T	T	T	T	T

C. Biconditional statement: logically equivalent statements

1. Definition of a biconditional statement — HPR...68-70

2. Form of a biconditional statement

a. p if and only if q
b. p implies q and q implies p: $p \longleftrightarrow q$

3. Truth table for the biconditional statement (optional)

p	q	$p \longleftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

4. Proving the contrapositive law

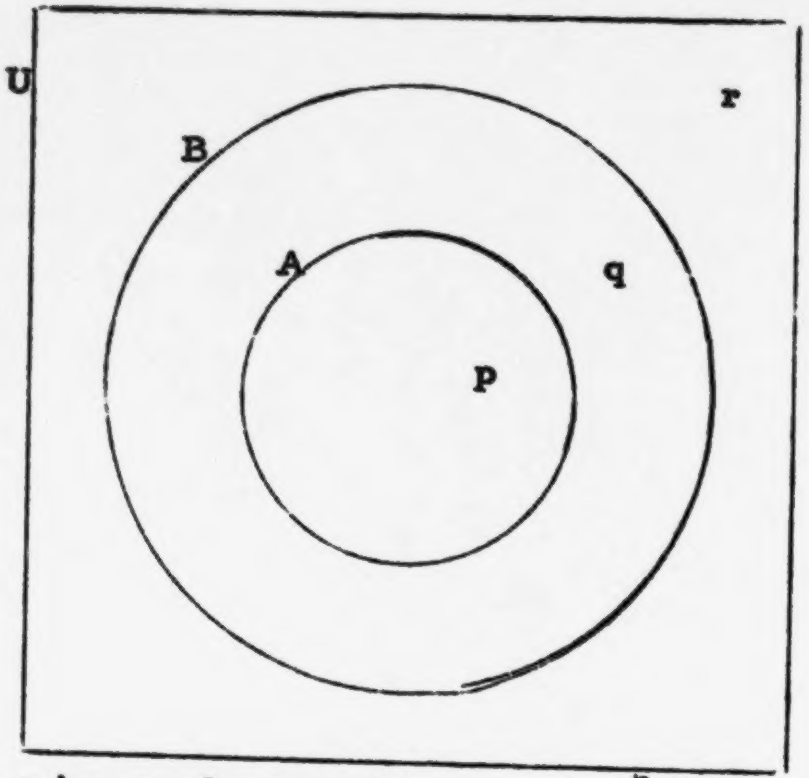
   a. Truth table (optional)

      (The student should compare the truth values for the variant forms of a conditional statement and discover that the truth values for the statement and its contrapositive are identical. They should also note that the inverse of a statement is the contrapositive of the converse.)

$p \longrightarrow q$	$\longleftrightarrow$	$\sim q \longrightarrow \sim p$
T	T	T
F	T	F
T	T	T
T	T	T

b. **Set diagram**

SMSG. . 241-243



p is any element of A

q is any element of B

$A \subseteq B$

$\therefore p \longrightarrow q$

$r \notin B;\ r \neq q$

$A \subseteq B$

$\therefore r \notin A;\ r \neq p$

$\therefore \sim q \longrightarrow \sim p$

[The contrapositive law states that a conditional statement and its contrapositive are logically equivalent:
$(p \longrightarrow q) \longleftrightarrow (\sim q \longrightarrow \sim p)$.]

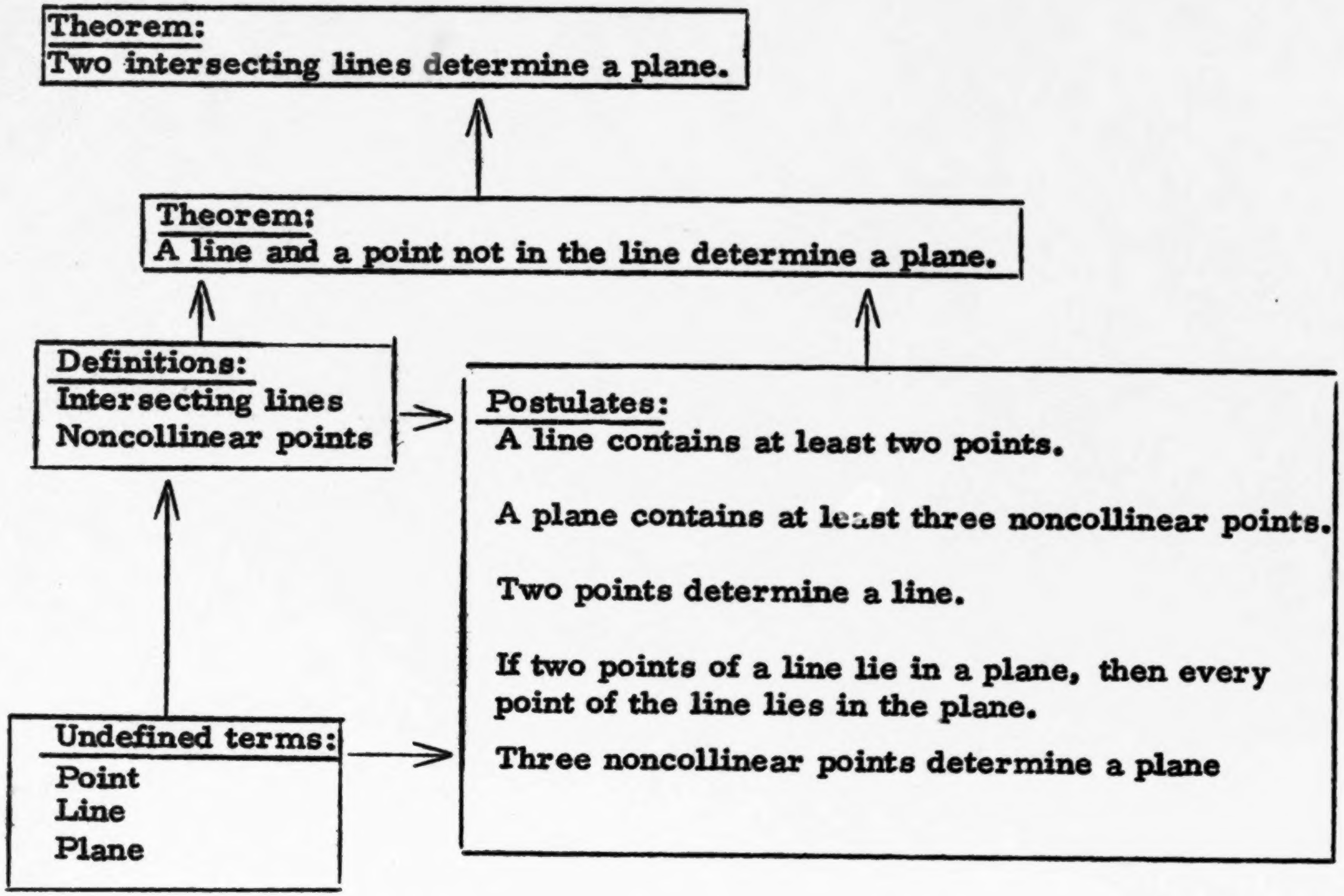
SCOPE	REFERENCES	
	CODE	PAGES
5. Definitions as biconditional statements		
a. A definition can be written in "if and only if" form.		
b. A definition has two parts.	SMSG..	241-243
c. A definition is reversible.		
(The teacher should give students practice in writing definitions of previously defined terms in "if and only if" form.)		
II. Proving conditional statements	HPR...	72-79
A. Universal statements		
1. Definition of a universal statement		
2. Definition of a counterexample of the universal statement		
3. The use of a counterexample to disprove a universal statement		
B. Reasoning patterns		
1. Asserting the antecedent: correct reasoning		
2. Asserting the consequent: incorrect reasoning	HPR...	133-134
III. Two types of reasoning		
(The filmstrip Nature of Proof, McGraw-Hill, will be helpful in introducing this section.)		
A. Inductive reasoning	SMSG..	4-9
1. Based on experiment and observation	MZ....	51-55

SCOPE	REFERENCES	
	CODE	PAGES
2. Useful in verifying the truth value of a universal statement if the replacement set is a finite set consisting of a small number of elements.		
3. Cannot be used to verify the truth value of a universal set if the replacement set is an infinite set		
B. Deductive reasoning		
1. Verifies the truth value of a universal statement by means of a logical argument based on undefined terms, definitions, and statements that are accepted as true without proof, and previously proved statements	SMSG. . MZ. . . .	10-15 56-59
2. Verifies the truth value of the statement for each element of the replacement set		
IV. Essentials of a mathematical system		
A. Undefined terms		
B. Definitions		
C. Postulates	JDD. . .	104-106
1. Definition: A postulate is a statement accepted as being true without proof.	SMSG. .	36-45

SCOPE	REFERENCES	
	CODE	PAGES
2. Postulates from Unit One and Unit Two a. A line contains at least two points; a plane contains at least three points not all in one line; and space contains at least four points not all in one plane. b. Two points determine a line. c. If two points of a line lie in a plane, then every point of the line lies in the plane. d. Three noncollinear points determine a plane. e. If two planes intersect, their intersection is a line. D. Theorems 1. Definition: A theorem is a universal statement that has been proved. 2. Theorems from Unit Two a. If two lines intersect, then their intersection is a single point. b. A line and a point not in the line determine a plane. c. Two intersecting lines determine a plane.		

SCOPE	REFERENCES	
	CODE	PAGES

(The following diagram should help students to understand the role of undefined terms, definitions, postulates, and theorems in proof.)



Theorem:
Two intersecting lines determine a plane.

Theorem:
A line and a point not in the line determine a plane.

Definitions:
Intersecting lines
Noncollinear points

Postulates:
A line contains at least two points.

A plane contains at least three noncollinear points.

Two points determine a line.

If two points of a line lie in a plane, then every point of the line lies in the plane.

Three noncollinear points determine a plane

Undefined terms:
Point
Line
Plane

SCOPE	REFERENCES CODE	 PAGES
V. **Properties of real numbers used in geometry**		
(The teacher should give many numerical examples illustrating the properties of equality in daily drills over an extended period of time rather than work for complete mastery of the topic in a few days.)		
A. Review of the basic laws of real numbers for addition and multiplication	HPR...	86-97
1. Closure property		
2. Commutative property	JDD...	99-103
3. Associative property		
4. Distributive property		
5. Identity element	MZ...	60-63
	SMSG.	233-237
a. Principle of zero		
b. Principle of one		
6. Inverse element		
a. Additive inverse		
(If $x + b = a$, then $x = a - b$.)		
b. Multiplicative inverse		
(If $b \neq 0$ and $xb = a$, then $x = \frac{a}{b}$.)		

SCOPE

REFERENCES

CODE | PAGES

B. Properties of equality of real numbers

1. Reflexive property: For any a, $a = a$.
2. Symmetric property: If $a = b$, then $b = a$.
3. Transitive property: If $a = b$ and $b = c$, then $a = c$.
4. Substitution property: If $a = b$, then a may be substituted for b in any expression.

5. Addition property: If $a = c$ and $b = d$, then:

   a. $a + b = c + d$.

   (If equals are added to equals, the sums are equal.)

   b. $a - b = c - d$.

   (If equals are subtracted from equals, the differences are equal.)

6. Multiplication property: If $a = c$ and $b = d$, then:

   a. $ab = cd$.

   (If equals are multiplied by equals, the products are equal.)

   b. $\frac{a}{b} = \frac{c}{d}$, if b and d are nonzero numbers.

   (If equals are divided by nonzero numbers that are equal, the quotients are equal.)

SCOPE	REFERENCES	
	CODE	PAGES
C. Properties of order of real numbers	SMSG	63-64
1. Linear property: If a and b are two given distinct real numbers, then either $a > b$ or $b > a$, but not both.	HPR	113-114
2. Definition of betweenness: If a, b, c are distinct real numbers, then b is between a and c if and only if $a > b$ and $b > c$ or $a < b$ and $b < c$.		
3. Definition of positive: A real number p is positive if and only if $p > 0$.		
4. Definition of negative: A real number p is negative if and only if $p < 0$.		
5. Transitive property: If $a > b$ and $b > c$, then $a > c$.		
6. Additive property: If $a > c$ and $b > d$, then $a + b > c + d$.		
7. Multiplicative property: If $a > b$ and $c > 0$, then $ca > cb$.		
VI. Correspondence and betweenness		
A. Informal development		
1. One-to-one correspondence between the set of points on a line and the set of real numbers	HPR	105-107
a. The number line	JDD	14-17
b. Coordinate of a point		
c. Graph of a number		

SCOPE	REFERENCES	
	CODE	PAGES
2. One-to-one correspondence between the set of rays with a common endpoint in a closed half-plane and the set of real numbers from 0 to 180 inclusive		
a. Family of rays		
b. Half-family of rays	HPR	111-112
c. Coordinate of a ray		
B. Formal development		
1. Definitions		
a. Betweenness preserving correspondence		
b. Coordinate system	HPR	108-110
c. Absolute value of a number: For any real number x,	JDD	18-21
if $x \geq 0$, then $\mid x \mid = x$;		30-36
if $x < 0$, then $\mid x \mid = -x$.	HPR	116-117
	HPR	125
(Students should make use of absolute value to determine	JDD	108
the distance between two points when the coordinates of	HPR	124
the points are given and the measure of an angle when the coordinates of the two rays are given.)		
2. Postulates		
a. Ruler postulate		
b. Protractor postulate		

SCOPE	REFERENCES	
	CODE	PAGES
c. Betweenness distance postulate: If A, B, C are distinct points such that B is between A and C, then m(AB) + m(BC) = m(AC).		
d. Betweenness angles postulate: If $\overrightarrow{OA}$, $\overrightarrow{OB}$, and $\overrightarrow{OC}$ are distinct rays such that $\overrightarrow{OB}$ is between $\overrightarrow{OA}$ and $\overrightarrow{OC}$, then $m\angle AOB + m\angle BOC = m\angle AOC$.	HPR... JDD...	125-128 125
(Postulates c and d may be proved and referred to as theorems rather than postulates. The use of the traditional statement "The whole is equal to the sum of its parts" should be discouraged. See Fundamentals of Mathematics by Richardson, chapter 15, for a discussion of the postulate and why it should not be used in proof.)		
VII. Postulates relating to triangles		
A. Congruence postulates		
1. SAS 2. ASA 3. SSS	HPR...	124
B. Properties of congruence of triangles		
1. Reflexive property: For any triangle, $\Delta ABC \cong \Delta ABC$. 2. Symmetric property: If $\Delta ABC \cong \Delta DEF$, then $\Delta DEF \cong \Delta ABC$. 3. Transitive property: If $\Delta ABC \cong \Delta DEF$ and $\Delta DEF \cong \Delta XYZ$, then $\Delta ABC \cong \Delta XYZ$.	SMSG..	233-234
C. For each triangle, if a line passes through a vertex of the triangle and is between two sides of the angle at this vertex, then the line intersects the opposite side of the triangle between its endpoints.	HPR...	129

SCOPE

UNIT FOUR

INTRODUCTION TO FORMAL PROOF

(Time allotment - 17 days)

In this unit the student will present formal proofs using principles which have been introduced previously. The proof in this text follows the traditional two column style. This parallel treatment of statements and reasons is a valuable pedagogical device for two reasons:

(1) The two-column form helps the student to understand that each assertion he makes must be supported by a definition, postulate, or previously proved theorem.

(2) The two-column form expedites checking and evaluation of formal proof.

Mathematicians frequently write proofs in paragraph form. Students should be made aware of this type of proof. They should be required to present both written and oral proofs in paragraph style.

I. The plan of a formal proof

   A. Representation of a theorem by a drawing

   B. Statement of hypothesis and conclusion in terms of the drawing

   C. Use of analysis to discover a plan of proof

REFERENCES
CODE | PAGES
HPR...142
JDD...71

SCOPE

REFERENCES

CODE | PAGES

D. Synthetic presentation of a proof

II. Elementary theorems pertaining to pairs of angles

(The teacher should remind the student he can prove two angles are equal by:

1. proving they have the same measure;
2. proving they are corresponding parts of congruent triangles.

As the course progresses he will be able to add to this list. The student should be encouraged to keep this list available, as he may wish to make reference to it.)

A. If two lines intersect, then the vertical angles are equal.

B. If two angles are equal, then their supplements are equal. (An alternate statement of the theorem is: Supplements of equal angles are equal.)

C. If two angles are equal, then their complements are equal. (An alternate statement of the theorem is: Complements of equal angles are equal.)

D. An angle is a right angle if and only if the sides of the angle are perpendicular.

SCOPE	REFERENCES	
	CODE	PAGES
E. Two adjacent angles form a linear pair if and only if they are supplementary.		
(This will provide an opportunity for the student to prove theorems stated in the biconditional form. In this stage of the student's development it may be necessary to remind him there are two parts to the proof of theorems given in this manner.)		
III. Proofs of constructions		
A. Basic assumptions relating to the use of geometric instruments	JDD. . .	359-360
1. A straight edge can be used to draw a line. 2. A compass can be used to mark off equal segments.		
B. Review of postulates		
1. Two points determine a line. 2. Congruence postulates of triangles		
C. Constructions to be proved		
(The presentation of "proofs" of constructions may be criticized on the grounds that nothing is really proved. Since constructions are a physical representation of geometrical ideas rather than a mathematical concept, these constructions are not necessary for a logical development of geometry. However, these proofs provide the student with practice in writing easy proofs of tangible rather than an abstract statement.)		

SCOPE	REFERENCES	
	CODE	PAGES
IV. More difficult proofs	HPR...	148-150
(Many students have difficulty with proofs involving overlapping triangles. The teacher should make use of colored chalk, cross-hatching, and the overhead projector in the presentation of overlapping triangles. Requiring students to outline the triangles in different colors is helpful in planning the proof.)		
V. The use of auxiliary lines in proof	MZ....	110-111
(The term "auxiliary line" is used to indicate a line that will be represented in the drawing to facilitate the understanding of a proof. The teacher should point out to the student that these lines are not constructed since they already exist. These lines are not indicated by the theorem and therefore are not part of the original drawing.)		
A. The need for auxiliary lines		
B. Restrictions on the use of auxiliary lines		
1. Conditions that determine a line 2. Overdetermined lines 3. Underdetermined lines		
VI. Theorems relating to sides and angles of a triangle		
A. If two sides of a triangle are equal, then the angles opposite these sides are equal.	HPR...	152

SCOPE	REFERENCES	
	CODE	PAGES
B. If two angles of a triangle are equal, then the sides opposite these angles are equal.	HPR...	157
(The teacher may wish to present alternate proofs for the two theorems stated above. These proofs are to be found on pages 276 and 282 of SMSG.)		
C. An exterior angle of a triangle is greater than either nonadjacent angle of the triangle.	HPR... TM ...	162 21
D. If two sides of a triangle are unequal, then the angles opposite these sides are unequal in the same order.	HPR...	250
(The contrapositive law of theorem A proves the inequality but it does not prove the order.)		
E. If two angles of a triangle are unequal, then the sides opposite these angles are unequal in the same order.	HPR...	251
(The following corollaries may be delayed until unit VI or presented at this time:		
1. The measure of the hypotenuse of a right angle is greater than the measures of either of the other sides.		
2. The shortest line segment from a point to a line is the line segment from the point perpendicular to the line.)		

SCOPE

REFERENCES
CODE | PAGES

UNIT FIVE

PARALLEL AND PERPENDICULAR LINES AND PLANES

(Time allotment - 20 days)

The teacher should preview the filmstrip Parallel and Perpendicular Lines, McGraw-Hill, prior to this unit. Frames 1-14 provide an excellent introduction to the unit while the remaining frames are applicable to later sections. However, some teachers may prefer to use the entire filmstrip as a review of the unit rather than as a means of introducing concepts.

The teacher must give careful attention to the selection of exercises for assignments. Exercise 9 on page 186 and exercises 19, 20, 22 on page 194 of the basic text are usually treated as theorems. The exercises should be proved and used as theorems.

A great deal of time could be devoted to the proofs of the exercises on page 220 of the basic text. However, presenting a few proofs formally and the remaining exercises informally would be a more judicious use of time.

The student should use every means at his disposal to develop an understanding of spacial relations, including the use of 5 × 3 cards and toothpicks to represent planes and lines respectively. With these materials the student can represent these relations by means of "models." Students should also make drawings of three-dimensional figures. A short summary of drawing space figures can be found on pages 913 through 919, Appendix V, SMSG.

The teacher can help develop basic space concepts by using large models, preferably student-made, and overlays with the overhead projector.

SCOPE	REFERENCES CODE	 PAGES
I. Parallel lines		
A. Basic concepts		
1. Physical concepts		
2. Geometrical concepts		
B. Definitions	HPR...	168-169
1. Parallel lines		
2. Skew lines		
3. Transversal		
4. Related angles		
a. Alternate interior angles		
b. Alternate exterior angles		
c. Corresponding angles		
d. Consecutive interior angles		
C. Postulate: In a plane which contains a line m and a point p not on line m there is one and only one line through p parallel to m.		
II. Indirect proof	JDD...	163-166
A. Basic concepts	HPR...	179
1. Negative of a statement		
2. Counterexamples of a statement		

SCOPE	REFERENCES	
	CODE	PAGES
B. Examples of indirect proof		
1. Nongeometrical 2. Geometrical		
C. Procedure for indirect proof		
1. List all possibilities. 2. Assume the negative of the proposed conclusion. 3. Prove the selected possibility contradicts the given information, a definition, a postulate, or a previously proved theorem. 4. Repeat the procedure until only the desired conclusion remains.		
D. Theorems involving indirect proof		
1. If lines a, b, and c are all in the same plane and a || b and b || c, then a || b. (Transitive property of parellel lines)	HPR	176
2. If a line is in the plane of two parallel lines and intersects one of the parallel lines, then it intersects the other also.	HPR	189
3. If two lines in the same plane are intersected by a transversal forming equal alternate interior angles, then the lines are parallel.	HPR	180-181
a. Corollary: If two lines in the same plane are intersected by a transversal so that two corresponding angles are equal, then the lines are parallel.		

SCOPE	REFERENCES	
	CODE	PAGES
b. Corollary: If two lines in the same plane are intersected by a transversal so that two alternate exterior angles are equal, then the lines are parallel.		
c. Corollary: If two lines in the same plane are intersected by a transversal so that two consecutive interior angles are supplementary, then the lines are parallel.		
d. Corollary: If each of two lines in a plane is perpendicular to the same line, then the lines are parallel.		
4. If two parallel lines are intersected by a transversal, then the alternate interior angles are equal.	HPR...	185-186
a. Corollary: If two parallel lines are intersected by a transversal, then the corresponding angles are equal.		
b. Corollary: If two parallel lines are intersected by a transversal, then the alternate exterior angles are equal.		
c. Corollary: If two parallel lines are intersected by a transversal, then the consecutive interior angles are supplementary.		
d. Corollary: If a line is in the plane of two parallel lines and is perpendicular to one of the parallel lines, then it is perpendicular to the other also.	HPR...	189
(The teacher should use the contrapositive of theorems 2 and 3 to establish other theorems.	HPR...	195

SCOPE	REFERENCES CODE	REFERENCES PAGES
At this time the students should summarize:		
(1) the conditions which must be satisfied for two lines to be parallel, and		
(2) the conditions satisfied when two lines are parallel. These conditions are summarized on pages 382 and 391 of SMSG.)		
III. Theorems and corollaries relating to the sum of the measure of a triangle.	HPR...	196-197
A. Basic theorem: The sum of the measures of the angles of a triangle is 180°.		
B. Corollaries and related theorems		
1. The measure of an exterior angle of a triangle is equal to the sum of the measures of the nonadjacent angles of the triangle.		
2. The sum of the measure of the angles of a four-sided figure is 360°.		
3. If two angles of one triangle are equal to two angles of another triangle, then the third angles are equal.		
4. If two triangles have two angles and a side of one triangle equal to two angles and a side of the other triangle, then the triangles are congruent. (AAS)		

SCOPE	REFERENCES CODE	REFERENCES PAGES
5. If two right triangles have the hypotenuse and an acute angle of one equal to the hypotenuse and acute angle of the other, then the triangles are congruent. (HA)		
6. If two right triangles have the hypotenuse and a side of one equal to the hypotenuse and side of the other, then the triangles are congruent. (HS)	HPR...	200
7. If the measures of one of the acute angles of a right triangle is 30°, then the measure of the side opposite this angle is equal to one-half the measure of the hypotenuse.		
IV. Perpendiculars in a plane		
A. Theorem: At a point on a line in a given plane, there is one and only one line perpendicular to the given line.	HPR...	204-205
B. Corollaries		
1. In the plane of a line and a point not on the line, there exists one and only one perpendicular to the line through the point.		
2. If in a plane a point is equidistant from the endpoints of a line segment, the point lies on the perpendicular bisector of the line segment.		
3. If in a plane each of two points is equidistant from the endpoints of a line segment, they determine the perpendicular bisector of the line segment.		

SCOPE	REFERENCES	
	CODE	PAGES
V. Postulates and theorems relating to planes		
A. Postulates		
1. Three noncollinear points determine a plane. 2. If two points of a line lie in a plane, then every point of the line lies in the plane.	HPR...208	
3. If two distinct planes have a point in common, then they have a line in common.		
4. A line either is parallel to a plane, intersects it in one point, or lies completely in the plane.		
(A line parallel to a plane is undefined. However, the teacher may wish to define the term at this time.)		
B. Theorems		
1. If a line not in a plane intersects the plane, the intersection is exactly one point.	HPR...210	
2. Two parallel lines determine a plane.		
VI. Perpendiculars in space	HPR...211-212	
A. Definitions		
1. A line perpendicular to a plane 2. An oblique line to a plane 3. Perpendicular planes		

SCOPE	REFERENCES	
	CODE	PAGES
B. Theorems		
1. If a line intersects a plane and is perpendicular to two lines in the plane at their intersection, then it is perpendicular to the plane.	HPR...	214-216
2. Two lines perpendicular to the same plane are parallel.		
C. Corollaries		
1. At a point on a line, there is one and only one plane perpendicular to the line.		
2. All the perpendiculars to a line through a point on that line lie in one plane, and that plane is perpendicular to the line.		
3. Through a point not on a line, there is one and only one plane perpendicular to the line.		
4. At a point in a plane, there is one and only one line perpendicular to the plane.		
5. From a point not in a plane there is one and only one line perpendicular to the plane.		
VII. Parallel planes	HPR...	220
A. Definitions		
1. A line parallel to a plane 2. Two parallel planes		

SCOPE

REFERENCES

CODE | PAGES

B. Theorems

1. If two planes are parallel to a third plane, they are parallel to each other. (Transitive property of parallel planes)

2. If two parallel planes are intersected by a third plane, the lines of intersection are parallel.

3. Two planes perpendicular to the same line are parallel.

4. If two lines are parallel, every plane containing one of the lines and only one, is parallel to the other line.

VIII. Concepts of duality relations between two and three dimensional statements

A. Extensions by intuition of two dimensional statements to three dimensional statements by various replacements of "line" or "lines" by "plane" or "planes"

B. Some theorems which have duality relations

1. If two lines (planes) are parallel to the same line (plane), they are parallel to each other.

2. If a line (plane) intersects one of two parallel lines (planes), it intersects the other also.

SCOPE	REFERENCES	
	CODE	PAGES
3. **If two lines (planes) are perpendicular to the same plane (line), they are parallel.** **(The teacher should have the students recall other theorems which have a duality relation.)** **C. Some theorems which do not have duality relations** 1. **If two planes are parallel to the same line, they are parallel.** 2. **If two planes are perpendicular to the same plane, they are parallel.**		

SCOPE

REFERENCES

CODE PAGES

UNIT SIX

SIMILARITY

(Time allotment - 20 days)

The purpose of this unit is to develop the basic properties of proportion and similarity which lead to the proof of the Pythagorean Theorem. This is followed by a discussion of the properties of special right triangles. These properties which are related to the Pythagorean Theorem should be stressd since they are useful in the solution of many practical problems. They also provide a basis for the development of many topics in subsequent courses in mathematics, such as trigonometry.

The last part of the chapter "Similarity" in the basic text deals with inequality relations in triangles. These relations were developed in Unit Four following the isosceles triangle theorems. For this reason they are omitted from the outline in this unit. However, it would be desirable for the teacher to review these concepts by including them in daily drills.

Before teaching the unit, the teacher should preview the following filmstrips:

Similar Triangles - Experiment and Deduction, Film Strip of-the-Month Club.
The Pythagorean Theorem, Film Strip of-the-Month Club.
Similarity, McGraw-Hill Company.

It is suggested that the first two filmstrips be used in the development of the unit, and that the last filmstrip be used as a summarizing activity for the unit.

SCOPE	REFERENCES	
	CODE	PAGES
The teacher should note that in the McGraw-Hill filmstrip the definition of similar figures must be revised to include congruent figures. This can be done by emphasizing that similar figures are the same shape but not necessarily the same size.		
I. Similar figures		
A. Physical concept		
1. Scale drawings 2. Same shape but not necessarily the same size	SMSG..	438-439
B. Corresponding parts of similar polygons		
1. Corresponding sides 2. Corresponding angles		
II. Ratio and proportion		
A. Definition		
1. Ratio	HPR...	225-226
2. Proportion 3. Constant of proportionality	JDD...	229-232
B. Methods of writing a proportion	SMSG..	440
1. $\frac{a}{b} = \frac{c}{d}$		
2. $(a, b) \overset{=}{p} (c, d)$	SMSG..	439-448

(An alternate approach to the study of proportion uses ordered pairs. This method has many distinct mathematical advantages. In addition, the teacher will find the notation easier to use in typing. A description of the use of ordered pairs may be found on pages 439-448 of SMSG.)

C. Terms of a proportion

1. First, second, third, fourth
2. Means, extremes

D. Properties of a proportion for nonzero numbers

(The following properties are always true if a, b, c, d $\neq$ 0. However, some of the properties are true if one or more of the numbers is 0. For example, if a=c =0, b=3, d= 4, then the product property is true because:

(1) the two ratios are equal: $\frac{0}{3} = \frac{0}{4}$

(2) the product of the means is equal to the product of the extremes: $0 \cdot 4 = 0 \cdot 3$

However, this property would not be true if a = 3, c = 4, b = d = 0 because division by zero is impossible. Similar numerical examples can be used as illustrations. The teacher teacher should always stress that examples which imply division by zero must be excluded.

SCOPE

1. **Alternation property:** If $\frac{a}{b} = \frac{b}{d}$, then $\frac{a}{c} = \frac{b}{d}$.

2. **Inversion property:** If $\frac{a}{b} = \frac{c}{d}$, then $\frac{b}{a} = \frac{d}{c}$.

3. **Equal numerators (denominators) property:** If $\frac{a}{b} = \frac{c}{d}$, then $a = c$ if and only if $b = d$.

4. **Product property:** If $\frac{a}{b} = \frac{c}{d}$, then $ad = bc$.

5. **Addition property:** If $\frac{a}{b} = \frac{c}{d}$, then $\frac{a+b}{a} = \frac{c+d}{c}$.

6. **Sequence property:** If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \ldots = \frac{m}{n} = k$, then

$$\frac{a + c + e + \ldots + m}{b + d + f + \ldots + n} = k.$$

[If $\frac{a}{b} = \frac{c}{d}$, the terms a, b, c, and d form a proportion, and the numbers (a, b) are said to be proportional to the numbers (c, d). A sequence of more than two equal ratios such as $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \ldots = \frac{m}{n} = k$ is not called a proportion. However, the numbers (a, c, e, ..., m) are said to be proportional to the numbers (b, d, f, ..., n). The number k is called the constant of proportionality.]

REFERENCES

CODE	PAGES
HPR...	226-227
JDD...	234-235
MZ...	263-266
SMSG..	444-446

SCOPE	REFERENCES	
	CODE	PAGES
III. Similar triangles		
A. Definition of similar triangles	HPR...	228-230
B. Notation: $\triangle ABC \sim \triangle XYZ$	SMSG..	450
1. There is a one-to-one correspondence between the vertices of two similar triangles.		
$A \longleftrightarrow X$ $\quad B \longleftrightarrow Y$ $\quad C \longleftrightarrow Z$		
2. Corresponding angles of two similar triangles have the same measure.		
$m\angle A = m\angle X$ $\quad m\angle B = m\angle Y$ $\quad m\angle C = m\angle Z$		
3. Corresponding sides of two similar triangles are proportional.		
$\frac{AB}{XY} = \frac{BC}{YZ} = \frac{AC}{XZ} = k$		
C. Properties of similar triangles	HPR...	228-232
1. Reflexive property: $\triangle ABC \sim \triangle ABC$. 2. Symmetric property: If $\triangle ABC \sim \triangle XYZ$, then $\triangle XYZ \sim \triangle ABC$. 3. Transitive property: If $\triangle ABC \sim \triangle DEF$ and $\triangle DEF \sim \triangle XYZ$, then $\triangle ABC \sim \triangle XYZ$.	SMSG..	453
D. Methods of proving triangles similar		
(The filmstrip Similar Triangles - Experiment and Deduction, Film Strip of-the-Month Club, can be used	HPR...	232-237

SCOPE	REFERENCES	
	CODE	PAGES

to introduce this section.)

1. SAS postulate for similarity: Two triangles are similar if an angle of one triangle is equal to an angle of the other triangle and the ratios between each pair of corresponding sides that include this angle are equal.

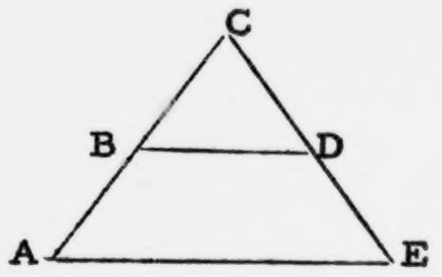
2. AA theorem for similarity: Two triangles are similar if two angles of one triangle are equal to two angles of the other triangle.

3. SSS theorem for similarity: If the measures of the corresponding sides of two triangles are proportional, the triangles are similar.

(It is usually difficult for the slower student to establish proportions from similar triangles. While it is important for all students to use the correspondence to help them, certain students will need an additional device. Such a device might be incorporated in the body of the proof with very little difficulty. For example:

Given: $\overline{BD} \parallel \overline{AE}$

Prove: $\frac{AC}{BC} = \frac{EC}{DC}$



SCOPE	REFERENCES CODE | PAGES

Statements:

1. $\overline{BD} \parallel \overline{AE}$

In $\triangle ACE$ and $\triangle BCD$

2. CE | $\angle CAE = \angle CBD$ | CD
3. CA | $\angle CEA = \angle CDB$ | CB
4. AE | $\angle ACE = \angle BCD$ | BD
5. $\therefore$ $\triangle ACE \sim \triangle BCD$
6. $\frac{CE}{CD} = \frac{CA}{CB} = \frac{AE}{BD}$

(The proof would begin in a normal fashion, except for the listing of the triangles which are involved. It is necessary to list all three pairs of equal angles. When similarity has been established, the student should go back and list the side opposite each angle of the appropriate triangle. The proportion is then readily established.)

E. Theorems and corollaries relating to similar triangles — HPR...232-233

1. Corollaries of the AA theorem

   a. A line parallel to one side of a triangle and intersecting the other two sides forms two similar triangles.

   b. If two isosceles triangles have one pair of corresponding angles equal, the triangles are similar.

SCOPE	REFERENCES	
	CODE	PAGES
c. If two right triangles have one pair of acute angles equal, the triangles are similar.		
d. All equilateral triangles are similar.		
2. Related theorems	HPR...	237-241
a. If a line intersects two sides of a triangle and is parallel to the third side, the measures of the corresponding line segments formed are proportional.		
Corollary A: If three or more parallel lines intersect two transversals, then the measures of the corresponding segments are proportional.		
Corollary B: If three or more parallel planes intersect two lines, then the measures of the corresponding segments are proportional.	MZ....	276
[A direct consequence of Corollary A and Corollary B would read as follows: If three or more parallel lines (planes) intersect two lines so that the measures of the segments on one line are equal, then the measures of the segments on the other line are equal.]		
b. Converse of a: If a line intersects two sides of a triangle and the measures of the corresponding line segments formed are proportional, then the line is parallel to the third side of the triangle.		
c. The bisector of an angle of a triangle divides the opposite side into segments whose measures are proportional to the measures of the sides of the angle of the triangle adjacent to the given segments.		

SCOPE

IV. The right triangle

A. Preliminary theorem and corollaries

REFERENCES CODE	 PAGES
HPR	242
SMSG	477-480
MZ	308-310

1. Theorem: The altitude to the hypotenuse of a right triangle forms two right triangles, each similar to the original triangle and similar to each other.

   (Problems 1-9 on page 242 of the text give the students practice in identifying corresponding sides and writing proportions of the measures of two pairs of corresponding sides for the similar triangles in the above theorem. Problems 10-13 on this page are sometimes stated as corollaries and used as reasons in the proof of the Pythagorean Theorem. Teachers will find it helpful to use cardboard models or transparencies with the overhead projector in teaching this theorem and the corollaries that follow.)

2. Corollary A (Problem 10): The square of the measure of the altitude of a right triangle is equal to the product of the measures of the segments on the hypotenuse.

   Corollary B (Problems 11 and 12): The square of the measure of either leg of a right triangle is equal to the product of the measures of the hypotenuse and the segment on the hypotenuse adjacent to that leg.

   Corollary C (Problem 13): In a right triangle the product of the measures of the hypotenuse and the altitude to the hypotenuse is equal to the product of the measures of the legs.

   [Problem 13 of the basic text should be: $m(AC) \cdot m(BC) = m(AB) \cdot m(CD)$]

SCOPE	REFERENCES	
	CODE	PAGES
B. The Pythagorean Theorem		
(The filmstrip The Pythagorean Theorem, Film Strip of-the-Month Club, gives the historical background, the proof, and applications to the theorem.)		
1. Historical background		
2. Theorem: In any right triangle, the square of the measure of the hypotenuse is equal to the sum of the squares of the measures of the other two sides.	HPR... JDD... SMSG..	243-247 272-273 480-485
3. Converse: If the square of the measure of one side of a triangle is equal to the sum of the measures of the other two sides, the triangle is a right triangle.		
(The teacher should give many numerical examples and practical applications of the Pythagorean Theorem.)		
C. Special right triangles	SMSG..	485-486
1. Pythagorean triples of numbers	JDD...	289-290
(Students should discover combinations of three numbers which can be used as measures of the sides of right triangles and be encouraged to use these combinations as ratio numbers in the solution of right triangle problems. Further investigation of this topic might be of interest to more capable students.)		

SCOPE

REFERENCES

CODE | PAGES

a. 3, 4, 5
b. 5, 12, 13
c. 8, 15, 17
d. 7, 24, 25
e. 1, 1, $\sqrt{2}$
f. 1, $\sqrt{3}$, 2

2. 30-60-90 triangles; 45-45-90 triangles

(The 30-60-90 triangles and the 45-45-90 triangles are widely used in the solution of practical problems in many related fields, such as industrial arts. The ratios of the measures of the sides of these two triangles are basic to the development of many important concepts in trigonometry. It is essential that students, especially those who continue work in mathematics, acquire a working knowledge of these ratios. It is suggested that (1) these ratios be derived from the Pythagorean Theorem, (2) that students receive immediate practice in their use through many numerical examples and problems dealing with practical applications of the triangles, and (3) that examples of a similar nature be included in daily drills over an extended period of time. This procedure should enable students to become thoroughly familiar with these ratios and their use. Since radicals are used in the solution of these problems, the teacher should include problems with radicals in daily drills for several days prior to the teaching of this section. Better students should be able to derive and make use of the following generalization: $\frac{1}{\sqrt{n}} = \frac{\sqrt{n}}{n}$.)

SCOPE	REFERENCES CODE	 PAGES
a. The ratio of the measures of the sides of a 30-60-90 triangle is $1, \sqrt{3}, 2$.	SMSG. JDD.	486-491 269-271
(1) The measure of the side opposite the 30 degree angle of a right triangle is equal to one-half the measure of the hypotenuse. (Theorem 5-7) $(s_{30} = \frac{h}{2})$		
(2) The measure of the side opposite the 60 degree angle of a right triangle is equal to one-half the measure of the hypotenuse multiplied by $\sqrt{3}$. $(s_{60} = \frac{h}{2}\sqrt{3})$		
b. The ratio of the measures of the sides of a 45-45-90 triangle is $1, 1, \sqrt{2}$.		
(1) The measure of the side opposite the 45 degree angle is a right triangle is equal to one-half the measure of the hypotenuse multiplied by $\sqrt{2}$. $(s_{45} = \frac{h}{2}\sqrt{2})$		
(2) The measure of the hypotenuse of an isosceles right triangle is equal to the product of $\sqrt{2}$ and the measure of one of the legs of the triangle.		

(Teachers may wish to include the following schematic device when summarizing this section. Students should find the sequence interesting as well as useful in recalling the relationships learned in this unit.

$$s_{30} = \frac{1}{2}h\sqrt{1} = \frac{1}{2}h$$

$$s_{45} = \frac{1}{2}h\sqrt{2}$$

$$s_{60} = \frac{1}{2}h\sqrt{3}$$

$$s_{90} = \frac{1}{2}h\sqrt{4} = h\ )$$

UNIT SEVEN

COORDINATE GEOMETRY

(Time allotment - 32 days)

This unit introduces the basic concepts of coordinate geometry, projection, and locus. In the basic text the section on coordinate geometry is followed by a rather rigorous treatment of projection and locus. In teaching this unit, it is best to relate the basic concepts of projection to coordinate geometry. Many of the theorems on projection should be treated informally or deleted. The theorems pertaining to locus should be treated informally with major emphasis on the meaning of locus and the application of the theorems pertaining to locus.

An alternate and more direct approach to the slope of parallel lines is presented in the outline. The slope of perpendicular lines is included so that students will have greater facility in proving theorems by coordinate methods.

A section on proof by coordinate methods is included at the end of the unit. The theorems which are listed are easy to prove by coordinate methods. The proofs of these theorems can be found in Appendices, Report of the Commission on Mathematics, pages 130-139.

The teacher should preview the following filmstrips before teaching the unit:

An Introduction to Coordinate Geometry, Film Strip of-the-Month Clubs.
The Slope of a Line, Film Strip of-the-Month Clubs.
Coordinate Geometry, McGraw-Hill Company.

SCOPE	REFERENCES	
	CODE	PAGES
The first two filmstrips could be used in the development of basic concepts and the third filmstrip would be useful as a summarizing activity for the unit.		
I. Introduction		
A. Work of René Descartes	HPR	261-262
B. Review of the basic concepts of coordinates on a line	JDD	434
II. Graphs in two dimensions		
(The material in this section will be a review for most students. The teacher should stress understanding and application of definitions and postulates rather than memorization.)		
A. Definitions 1. Orthogonal coordinate system 2. Axes 3. Origin 4. Graph of a number pair 5. Projection of a point on a line 6. Coordinates of a point a. Abscissa b. Ordinate	HPR SMSG	263-264 306;295 475, 501-510
(The abscissa is the number assigned to the projection of the point on the x-axis; the ordinate is the number assigned to the projection of the point on the y-axis.)		

SCOPE	REFERENCES	
	CODE	PAGES
7. Graph of an equation containing two variables		
8. Projection of a segment on a line		
B. Postulates		
1. There exists a one-to-one correspondence between the set of all ordered pairs of real numbers and the set of points in the plane. This is called a two dimensional coordinate system.		
2. The graph of every equation of the form ax + by = c, where x and y are the variables and a, b, and c are constants (and a and b are not both zero), is a straight line. Furthermore, every straight line is the graph of the solution set of some equation of the form ax + by = c.		
C. Theorems and corollaries	HPR...	268-269 296-297
1. The ordinate of each point on the x-axis is 0.		
2. For every line segment AB, if AB is parallel to OX, then m(AB) = $\lvert x_B - x_A \rvert$. ($\lvert x_B - x_A \rvert$ is the length of the projection of the segment AB on the x-axis.)		
III. Slope		
A. Physical concept	SMSG..	544-553
1. Steepness		
2. Rise divided by run		

SCOPE	REFERENCES CODE | PAGES

(Students should draw pictures of lines with slopes that are positive, negative, and equal to 0; they should understand that the slope of a vertical line is not defined.)

B. Formal development

1. Meaning of Δx, Δy
2. Theorem: The ratio of Δy to Δx is $\frac{-a}{b}$ for any two distinct points on the graph of $ax + by = c$ where $b \neq 0$.

HPR...272-277

3. Definition: If P and Q are any two distinct points on a nonvertical line, then the slope of the line is the ratio of $y_Q - y_P$ to $x_Q - y_P$. If $\Delta y = y_Q - y_P$ and $\Delta x = x_Q - x_P$, then the slope of the line is Δy divided by Δx.

   (The slope of a segment is equal to the ratio of the length of the projection of the segment on the y-axis to the length of the projection of the segment on the x-axis.)

4. If $\underline{l}$ is the graph of $ax + by = c$ and $b \neq 0$, then the slope of $\underline{l}$ is equal to $-\frac{a}{b}$.

5. The slope of a vertical line is not defined.

   (Division by zero is impossible.)

IV. Linear equations

A. Writing linear equations

HPR...279-280
286-288

SCOPE	REFERENCES	
	CODE	PAGES
1. General form: $ax + by = c$		
2. Slope-intercept form: $y = hx + k$ $(y = mx + b)$		
a. Slope: h (m)		
b. y-intercept: k (b)		
3. Two point form: $\frac{y - y_B}{x - x_B} = \frac{y_B - y_A}{x_B - x_A}$, if $x_B \neq x_A$		
4. Point-slope form: $y - y_B = h(x - x_B)$		
a. Slope: h		
b. Point: (x_B, y_B)		
B. Graphing linear equations		
C. Summarizing lesson		
(The following procedure may be used as an alternate approach to Section IV rather than a summarizing activity.)		
1. Use of similar triangles to prove that any two segments of a non-vertical line have the same slope	JDD...	414
(At this time students should understand that two distinct lines through the same point must have different slopes.)		

SCOPE	REFERENCES	
	CODE	PAGES
2. Deriving the two-point form for a linear equation		
3. Deriving the following forms for a linear equation from the two-point form:	SMSG..	559-563
a. Point-slope form b. Slope-intercept form c. General form		
V. Segments		
A. The distance formula to find the length of a segment	HPR...	280-293
B. The midpoint formula to find the coordinates of the midpoint of a segment		
C. The ratio formula to find the coordinates of a point which divides a segment into a given ratio (optional)		
(Drills should include numerical exercises which will assist students in discovering these formulas prior to deriving them. Following the derivation of the formulas the students should receive practice in applying these formulas in many numerical examples.)		
VI. Parallel and perpendicular lines		
A. Slope of parallel lines	JDD... HPR...	416 282-285
(The sequence of theorems and proofs given in the basic text should not be used; the use of similar triangles is more direct and easier to understand.)		

SCOPE	REFERENCES	
	CODE	PAGES
1. Two nonvertical lines are parallel if and only if they have the same slope. 2. Two nonvertical lines intersect if and only if their slopes are not equal. (This statement is true by the contrapositive law.)		
B. Slope of perpendicular lines (These proofs are not included in the basic text. However, they should be included in the course since they are basic theorems of coordinate geometry and will be used in proving theorems by coordinate methods.)	JDD SMSG	417 564-565
1. Two nonvertical lines are perpendicular if and only if the slope of one is the negative reciprocal of the slope of the other. (Alternate statement: Two nonvertical lines are perpendicular if and only if the product of their slopes is -1.)		
2. Two nonvertical lines are not perpendicular if and only if the slope of one line is not the negative reciprocal of the slope of the other. (Alternate statement: Two nonvertical lines are not perpendicular if and only if the product of their slopes is not equal to -1. This statement is true by the contrapositive law.)		
3. A horizontal line is perpendicular to a vertical line.		
C. Projection (optional) (The only theorem in this section that is used in the development of subsequent topics is Theorem 7, and this theorem can be proved by congruent triangles. The first six theorems	HPR	298-300

SCOPE	REFERENCES	
	CODE	PAGES
should be treated informally or omitted.)		
1. If a line segment AB has slope h, then the projection of AB on the y-axis equals h times the projection of AB on the x-axis.		
2. Two parallel line segments have equal projection on the x-axis if and only if they have equal projections on the y-axis.		
3. Two parallel line segments are equal if and only if their projections on the x-axis (y-axis) are equal.		
4. Parallel line segments between parallel lines are equal.		
D. Distance		
1. Definitions	HPR	...302
a. Distance from a point to a line b. Distance from a point to a plane c. Distance between two parallel lines		
2. Theorems		
a. If h and k are parallel lines and A and B are any points on h, then the distance from A to k equals the distance from B to k.		
(Alternate statement: Parallel lines are everywhere equidistant.)		
b. If h and k are parallel lines and A $\in$ h and B $\in$ k, then the distance from A to k equal the distance from B to h.		

SCOPE	REFERENCES	
	CODE	PAGES
VII. Proof by analytic methods		
A. Review of basic concepts in proof		
1. Use of the distance formula to prove segments are equal in length	JDD...	427-428
2. Use of the midpoint formula to prove a point or a line bisects a segment		
3. Use of the slope formula to prove two nonvertical lines are parallel (perpendicular)		
4. Use of simultaneous linear equations to determine the coordinates of the point of intersection of two lines		
B. Selecting a coordinate system		
1. Choosing the coordinate axes		
2. Determining an appropriate scale	JDD... SMSG..	437-442 572-577
(If the midpoint of a segment is needed in the solution of the problem, computation will be simplified if the coordinates of the endpoints are divisible by 2.)		
C. Theorems with easy analytic proofs		
(Students should be encouraged to write analytic proofs in paragraph form rather than in two column form.)	CEEB..	130-139
1. If a line segment joins the midpoints of two sides of a triangle, it is parallel to the third side and equal in measure to one-half the third side.	SMSG..	572-575

SCOPE	REFERENCES	
	CODE	PAGES
2. If a line bisects one side of a triangle and is parallel to a second side, it bisects the third side.		
3. The midpoint of the hypotenuse of a right triangle is equidistant from all three vertices of the triangle.		
4. Oblique line segments from a point in a perpendicular to a line are equal if and only if they cut off equal segments from the foot of the perpendicular.		
5. If from a point in a perpendicular to a line two unequal oblique line segments are drawn to the line, the longest segment cuts off the greater distance from the foot of the perpendicular.		
6. The medians to the two equal sides of an isosceles triangle are equal.		
7. In a plane, a point is in the perpendicular bisector of a line segment if and only if it is equidistant from the endpoints of the segment.	SMSG..	588
VIII. Locus		
A. Basic concepts		
1. A locus of points is a set of all points and only those points which satisfy one or more given conditions.	HPR...	300-301
2. A locus theorem is a statement of equivalence and can be expressed in "if and only if" form.		
3. The proof of a locus theorem consists of two parts:		
a. Every point of the locus satisfies the conditions.		
b. Every point that satisfies the conditions is in the locus.		

SCOPE	REFERENCES	
	CODE	PAGES
B. Locus theorems		
(The first theorem in this section was stated and proved as Theorem 7 in the previous section. Theorem 8 in this section should be proved by the synthetic method. All theorems except 1 and 7 in this section should be treated informally.)		
1. In a plane, the locus of points equidistant from two points is the perpendicular bisector of the segment determined by the two points.	HPR...	303-310
2. In space, the locus of perpendiculars to a line at a point in the line is the plane perpendicular to the line at that point.		
3. In space, the locus of points equidistant from two points is the plane which is the perpendicular bisector of the segment determined by the two points.		
4. In a plane, the locus of points equidistant from two parallel lines is the line parallel to each of the lines and midway between them.		
5. In a plane, the locus of points at a given distance from a line is a pair of lines parallel to the line and at the given distance from the line.		
6. In space, the locus of points equidistant from two parallel planes is the plane parallel to each of the two planes and midway between them.		
7. In space, the locus of points at a given distance from a plane is a pair of planes parallel to the plane and at a given distance from the plane.		

SCOPE	REFERENCES	
	CODE	PAGES
8. In a plane, the locus of points equidistant from the sides of an angle is the bisector of the angle.		
Corollary: In a plane, the locus of points equidistant from two intersecting lines is the pair of lines bisecting the angles formed by the lines.		
9. In space, the locus of points equidistant from the faces of a dihedral angle is the closed half-plane which bisects the dihedral angle.		
Corollary: In space, the locus of points equidistant from two intersecting planes is the pair of planes bisecting the dihedral angles formed by the given planes.		
C. Compound locus		
(An excellent method of helping students to visualize problems involving compound locus is to make separate overlays of the various loci described in a problem, and to use these overlays to illustrate the intersection of these loci.)	HPR...	311
D. Concurrent lines		
(Simple exercises in paper folding as illustrated in frames 14-22 of the filmstrip Properties of Triangles, McGraw-Hill, is an excellent method of leading the students to discover the theorems in this section. The chapter "Paper Folding" in the Eighteenth Yearbook of the National Council of Teachers of Mathematics gives excellent directions for this work.)		

SCOPE	REFERENCES	
	CODE	PAGES
1. Definition: The lines in a set are called concurrent lines if and only if there is exactly one point which lies in all of them.	SMSG..	589
2. Theorems (The theorems in this section, except for theorem b, can be proved by both analytic and synthetic methods. However, proof by analytic methods should be reserved for students who have a good background in algebra.)		
a. The perpendicular bisectors of the sides of a triangle are concurrent at a point equidistant from the vertices of the triangle. (Before proving this theorem, the teacher should review the fact that the locus of points equidistant from the end-points of a line segment is the perpendicular bisector of the segment.)	SMSG.. CEEB.. MZ....	589-590 135-139 470
b. The lines which contain the angle bisectors of the angles of a triangle are concurrent at a point equidistant from the sides of the triangle. (Before proving this theorem, the teacher should review the fact that the locus of points equidistant from the sides of an angle is the bisector of the angle.)	SMSG.. MZ....	591 471
c. The lines which contain the altitudes of a triangle are concurrent.	CEEB.. SMSG.. MZ....	135, 138 592-593 474

SCOPE	REFERENCES	
	CODE	PAGES
d. The medians of a triangle are concurrent at a point which is two-thirds of the distance from any vertex of the triangle to the midpoint of the opposite side.	CEEB. MZ. . .	135, 138 472
(Students should make wooden or cardboard models of triangles and draw the three medians of the triangle. These models can be used to illustrate the fact that the point of concurrency of the medians of a physical triangle is the center of gravity of the triangle.)		
IX. Coordinates in three dimensions (optional)	SMSG..	633-649
A. Basic postulate: There exists a one-to-one correspondence between the set of all ordered triples of real numbers and the set of points in space. This is called a three-dimensional coordinate system.		
B. An orthogonal-coordinate system		
1. x-axis 2. y-axis 3. z-axis 4. Origin		
C. Plotting points		
D. The distance formula		
E. The midpoint formula		

SCOPE	REFERENCES	
	CODE	PAGES
F. Theorems with analytic proofs		
1. From a point on a perpendicular to a plane, oblique line segments to the plane are equal if and only if they cut off equal line segments from the foot of the perpendicular.		
2. If two unequal line segments are drawn from a point in a perpendicular to a plane, the greater segment intersects the plane at a greater distance from the foot of the perpendicular.		
3. In space, the locus of points equidistant from two points is the plane which is the perpendicular bisector of the line segment determined by the two points.	SMSG.	. 650

SCOPE

REFERENCES
CODE | PAGES

UNIT EIGHT

POLYGONS AND POLYHEDRONS

(Time allotment - 20 days)

This unit presents the basic properties of polygons and polyhedrons with special emphasis on the properties of parallelograms. Since many of the theorems relating to parallelograms can be proved more directly by the use of coordinates, this method of proof is emphasized in the outline.

The section on polyhedrons is developed in an informal manner through the use of pictures, models, and experiments. In this section the teacher will note that the words "basic vocabulary and generalizations" have been used rather than "definitions and theorems."

I. Polygons

A. Definition

B. Separation: A polygon separates a plane into three distinct subsets.

HPR...318-319

1. The points in the polygon
2. The points in the interior of the polygon
3. The points in the exterior of the polygon

JDD...61-62

C. Parts of a polygon

MZ....215-217

1. Sides, consecutive sides
2. Vertices, consecutive vertices

SCOPE	REFERENCES	
	CODE	PAGES
3. Angles		
a. Interior angle		
b. Exterior angle		
4. Diagonal		
D. Nomenclature		
1. Types of polygons		
a. Convex		
b. Concave		
2. Polygons classified by number of sides		
a. Triangle		
b. Quadrilateral		
c. Pentagon		
d. Hexagon		
e. Heptagon		
f. Octagon		
g. Nonagon		
h. Decagon		
i. Dodecagon		
j. n-gon		

SCOPE	REFERENCES	
	CODE	PAGES
3. Special polygons		
a. Equilateral b. Equiangular c. Regular		
(The teacher should make use of models and set diagrams to illustrate the fact that an equilateral polygon is not necessarily equiangular.)		
E. Theorems relating to the angles of a polygon	SMSG..	728-735
1. The sum of the measures of the interior angles of a polygon is $180(n - 2)$.	MZ....	217-221
(This is problem 22, page 199 of the basic text; the following theorems are not included in the basic text.)		
2. The sum of the measures of the exterior angles of a polygon, one at each vertex, is 360.		
3. Each exterior angle of a regular polygon has a measure of $\frac{360}{n}$.		
4. Each interior angle of a regular polygon has a measure of $\frac{180(n-2)}{n}$ or $(180 - \frac{360}{n})$.		
II. Quadrilaterals		
A. Definition		

SCOPE	REFERENCES	
	CODE	PAGES
B. Parts of a quadrilateral	SMSG..	569-570 585
1. Vertices		
2. Angles, consecutive angles		
3. Sides		
a. Opposite sides		
b. Adjacent sides or consecutive sides		
4. Diagonal		
C. Nomenclature		
(The students should make set diagrams to show the relations that exist among the members of the set of quadrilaterals.)		
1. Parallelogram		
2. Rectangle		
3. Rhombus		
4. Square		
5. Trapezoid		
6. Isosceles trapezoid		
III. Parallelograms		
A. Basic theorems	HPR...	325-334
1. If two sides of a quadrilateral are equal and parallel, the quadrilateral is a parallelogram.		

SCOPE	REFERENCES	
	CODE	PAGES
2. If the opposite sides of a quadrilateral are equal, the quadrilateral is a parallelogram.		
3. The opposite sides of a parallelogram are equal in length.		
Corollary: A rhombus is an equilateral parallelogram.		
4. The consecutive angles of a parallelogram are supplementary.		
5. The opposite angles of a parallelogram have equal measure.		
Corollary: A rectangle is equiangular.		
B. Theorems relating to coordinates		
(Many theorems pertaining to parallelograms can be proved readily by coordinate methods if a coordinate system is properly chosen. The theorems that follow will assist the student in establishing such a system. The teacher may wish to show the filmstrip Geometric Proof Using Coordinates, Film Strip of-the-Month Clubs, as an introduction to this section.)	SMSG..	577-580
1. Given quadrilateral ABCD with A = (0, 0), B = (a, 0), D = (b, c), and C = (x, y), then ABDC is a parallelogram if and only if C = (a + b, c).		
2. If the coordinates of the vertices of a parallelogram are A = (0, 0), B = (a, 0), C = (a + b, c), and D = (b, c), then the parallelogram is a rectangle if and only if b = 0 .		

3. If the coordinates of the vertices of a parallelogram are $A = (0, 0)$, $B = (a, 0)$, $C = (a + b, c)$, and $D = (b, c)$ where $a > 0$, then the parallelogram is a rhombus if and only if $a = \sqrt{b^2 + c^2}$.

C. Additional theorems pertaining to parallelograms

(The teacher may wish to use the filmstrip Parallelograms and Their Properties, Film Strip of-the-Month Clubs, as an introduction to, or as a summary of, the section. The proofs of the following theorems should be done by the analytic method unless the synthetic method is specified. In order to contrast the analytic method and the synthetic method of proof, the teacher may wish to follow an analytic proof with an oral discussion of a plan for proving the same theorem by the synthetic method.)

1. A quadrilateral is a parallelogram if and only if the diagonals bisect each other.
2. A parallelogram is a rectangle if and only if the diagonals are equal in length.
3. A parallelogram is a rhombus if and only if the diagonals are perpendicular.
4. A parallelogram is a rhomus if and only if a diagonal bisects one of its angles. (Synthetic method)

SCOPE	REFERENCES	
	CODE	PAGES
D. Summary of properties	SMSG..	584
(The completion of the chart on page 584 of SMSG would be an excellent summarizing activity for students. The teacher should stress that the set of squares is the intersection of the set of rectangles and the set of rhombuses, and therefore a square would have the properties of the rectangle and the rhombus.)		
IV. Trapezoids		
A. Parts of a trapezoid		
1. Bases 2. Legs 3. Base angles 4. Median 5. Altitude		
B. Establishing a coordinate system for trapezoids	JDD...	452
1. Trapezoid 2. Isosceles trapezoid		
C. Proofs relating to trapezoids		
(Students should use analytic proof unless the synthetic method is specified.)		

SCOPE	REFERENCES	
	CODE	PAGES
1. The median of a trapezoid is parallel to the bases and is equal to one-half the sum of the measures of the bases. (The teacher should not use the analytic proof in the basic text.)	SMSG CEEB	586 135, 137
2. The diagonals of an isosceles trapezoid have the same measure. 3. The base angles of an isosceles trapezoid have equal measure. (Synthetic proof)	SMSG HPR	587 336
V. Summarization of properties of quadrilaterals (The teacher should find the following methods of value in summarizing the properties of quadrilaterals:		
(1) The filmstrip Sets of Quadrilaterals, McGraw-Hill Company (2) The completing of the chart on page 595 of SMSG (3) The inclusion in daily drills of exercises on quadrilaterals which have numerical coordinates assigned to their vertices	JDD SMSG SMSG	447-457 566 596-598
(4) The proof of the problems in this section. Students should use models and make drawings which will lead to the discovery of these statements.)	CEEB JDD	134, 137 67
A. The midpoints of the sides of any quadrilateral are the vertices of a parallelogram.		
B. The midpoints of the sides of any rectangle are the vertices of a rhombus.		

SCOPE	REFERENCES	
	CODE	PAGES
C. The midpoints of the sides of any rhombus are the vertices of a rectangle.		
VI. Polyhedrons (The content of this section should be treated informally. Models, pictures, and experiments should serve as the basis for building vocabulary and arriving at generalizations.)	MZ.... SMSG.. HPR... HPR...	240-243 744-789 410-413 416-417
A. Basic vocabulary 1. Polygonal surface or polygonal region 2. Polyhedron 3. Face 4. Edge 5. Vertex 6. Section 7. Polyhedral angle 8. Regular polyhedron		
B. Nomenclature 1. Types of polyhedrons a. Prism b. Truncated prism c. Pyramid d. Frustum of a pyramid e. Regular polyhedron		

SCOPE	REFERENCES	
	CODE	PAGES
2. Polyhedrons classified by number of faces (No polyhedron has less than four faces.) a. Tetrahedron b. Pentahedron c. Hexahedron d. Heptahedron e. Octahedron f. Nonahedron g. Decahedron h. Dodecahedron i. Icosahedron		
C. Generalizations (Teachers may wish to assign this section to a committee of students as a special report to the class. Many students will find the construction of stollated models of various polyhedrons an interesting project.)		
1. Euler's formula: $E + 2 = F + V$, where E = the number of edges, F = the number of faces, and V = the number of vertices.	SMSG..	777-779
2. The sum of the measures of any two face angles of a trihedral angle is greater than the measure of the third face angle.	SMSG..	780-788
3. The sum of the measures of all the face angles of any polyhedral angle is less than 360.		

SCOPE	REFERENCES	
	CODE	PAGES
4. There are exactly five types of regular polyhedrons. a. Tetrahedron--4 faces b. Hexahedron--6 faces c. Octahedron--8 faces d. Dodecahedron--12 faces e. Icosahedron--20 faces		

UNIT NINE

CIRCLES AND SPHERES

(Time allotment - 20 days)

Although the text presents circles and spheres in separate chapters, this approach is not recommended since the definitions and the basic concepts are so closely related. Circles and spheres are placed in separate sections of the outline only for ease in outlining. The recommended approach is to teach the definitions and basic concepts simultaneously. For example, if the students are asked to describe the set of points at a given distance from a given point, the resulting discussion should lead to the simultaneous development of the following terms:

(1) Circle: the set of all points in a plane a given distance from a given point.

(2) Sphere: the set of all points in space a given distance from a given point.

In discussing these definitions the students should understand that an immediate consequence of the definition of a circle and the definition of a sphere is that all radii of the same circle and all radii of the same sphere are equal.

Sections A through F on spheres parallel the development of sections A through F on circles, and the definitions, postulates, and theorems in these sections should be developed simultaneously. The remaining sections on spheres should be delayed until after the study of circles, and the concepts in these sections should be treated informally.

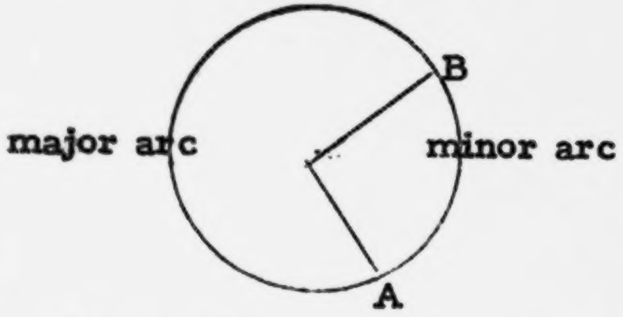
SCOPE	REFERENCES
	CODE PAGES
The work on cylinders in Chapter 9 of the basic text will be included in the last unit of the outline.	
I. Circles	
A. Basic definitions	
1. Circle	
2. Radius: outer end	HPR...350
3. Congruent circles	HPR...354
B. Separation: A circle separates a plane into three distinct subsets.	
1. The points of the circle	
2. The points in the interior of the circle: inside the circle	HPR...353
3. The points in the exterior of the circle: outside the circle	
C. Circles in a coordinate plane	HPR...350-352
1. Equation of a circle with center at the origin: $x^2 + y^2 = r^2$	
2. Equation of the circle with center at (h, k): $(x-h)^2 + (y-k)^2 = r^2$ (This is called the general equation of the circle.)	
D. Arcs of circles	HPR...355
1. Central angle	SMSG..840
2. Minor arc: If $\angle AOB$ is a central angle with A and B the points of intersection of the sides of the angle and the circle, and if $m(\angle AOB) < 180$,	

then the union of A, B, and all points of the circle in the interior of ∠AOB is a minor arc of the circle.

3. Major arc: If ∠AOB is a central angle with A and B the points of intersection of the sides of the angle and the circle, and if the m(∠AOB) < 180, then the union of A, B, and all points of the circle in the exterior of ∠AOB is a major arc of the circle.

   (The students should understand that if the term "arc" is used alone, it will refer to a minor arc.)

4. Semicircle: If the m(∠AOB) = 180, the points of the circle which lie in the same half-plane are called a semicircle.



5. Endpoints of the arc: The points A and B are called the endpoints of the arc.

E. Lines and segments relating to circles — HPR...359

1. Definitions

   a. Chord: A line segment whose endpoints are points of a circle is called a chord.

SCOPE	REFERENCES	
	CODE	PAGES
b. Diameter: A chord which contains the center of the circle is called a diameter.		
c. Secant: A line which intersects a circle in exactly two points is called a secant.		
d. Tangent: A line in the plane of the circle which intersects the circle in exactly one point is called a tangent. The point of intersection is called the point of tangency.	SMSG..	861-862
e. Tangent-segment: If the line QR is tangent to a circle at R, then the segment QR is a tangent-segment from Q to the circle.		
f. Secant-segment: If secant QR intersects a circle in A and B such that A is between Q and B, then QB is called the secant-segment from Q to the circle and QA is called the external secant-segment from Q to the circle.		
2. Basic concept: the tangent as the limiting position of the secant	JDD...	330-331
3. Postulate: If a line and a circle are in the same plane and the line contains one point in the interior of the circle, then the line intersects the circle in exactly two points.		
F. Relations of two circles and the lines and segments related to those circles		
1. Concentric circles		

2. Intersecting circles: two circles in the same plane with exactly two points in common

   a. Common chord
   b. Line of centers

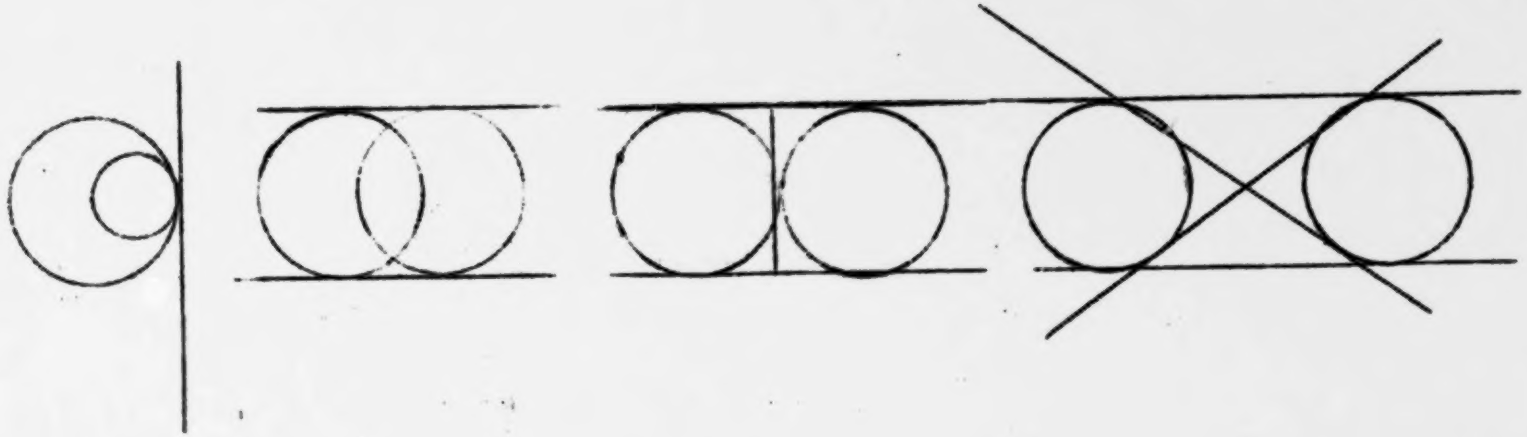
3. Tangent circles: circles in the same plane with exactly one point in common

   a. Internally tangent circles
   b. Externally tangent circles

4. Tangents to two circles

   a. Common tangent
   b. Common internal tangent
   c. Common external tangent

(The teacher should discuss the cases indicated by the figures below)



5. Summarizing activity

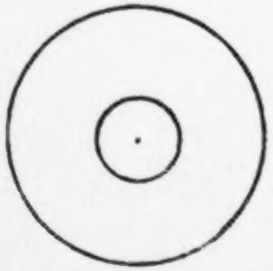
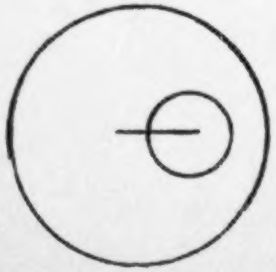
(The students should complete the chart below by:

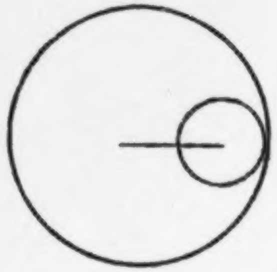
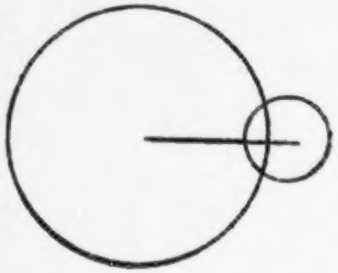
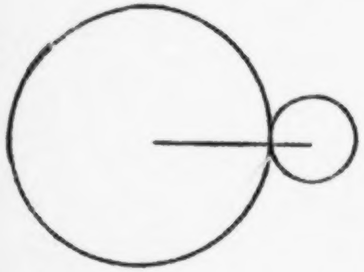
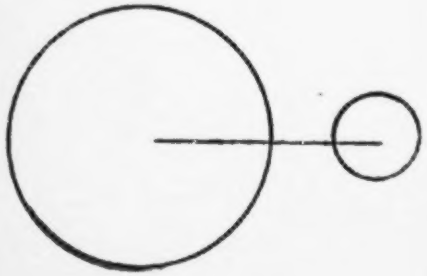
(1) Drawing the common tangents to the given circles in column 1 if these tangents exist,

(2) Writing the number of common tangents, the number of internally common tangents, and the number of externally common tangents in the appropriate column,

(3) Computing the length of the line of centers, d, if the radius of the larger circle is r_1 and the radius of the smaller circle is r_2.

The chart has been completed for the teacher.)

	Number of Common Tangents	Number of Common Internal Tangents	Number of Common External Tangents	Length of Line of Centers
	0	0	0	$d = 0$
	0	0	0	$d<(r_1 - r_2)$

SCOPE					REFERENCES	
					CODE	PAGES
	1	0	1	$d = (r_1 - r_2)$		
	2	0	2	$(r_1 - r_2) < d < (r_1 + r_2)$		
	3	1	2	$d = (r_1 + r_2)$		
	4	2	2	$d > (r_1 + r_2)$		

SCOPE	REFERENCES	
	CODE	PAGES
G. Measures of arcs		
1. Definitions		
a. The degree measure of an arc: If $\overset{\frown}{AXB}$ is any arc, then the degree measure of the arc denoted by $m(\overset{\frown}{AXB})$ is given as follows:	SMSG. . 841	
(1) If $\overset{\frown}{AXB}$ is a minor arc or semicircle, $m(\overset{\frown}{AXB})$ is the measure of the associated central angle.		
(2) If $\overset{\frown}{AXB}$ is a major arc, and $\overset{\frown}{AYB}$ is the corresponding minor arc, then $m(\overset{\frown}{AXB}) = 360 - m(\overset{\frown}{AYB})$.		
b. Equal arcs c. To bisect an arc d. Bisector of an arc	HPR. . . 356	
2. Theorem: If two central angles of a circle are equal, the arcs they intercept are equal in degree measure.		
Corollary: If the degree measures of two arcs are equal, the central angles which they intercept are equal.		
(Other theorems and corollaries are presented as exercises on pages 357 and 358 of the basic text. It is desirable to assign problems 1-8, 18-20, 25, and 26 so they can be used as theorems.)		

SCOPE	REFERENCES	
	CODE	PAGES
H. Theorems and corollaries pertaining to chords and tangents	HPR...	360-365
1. Theorems and corollaries pertaining to chords		
a. A diameter is the longest chord of a circle.		
b. If two chords of a circle are equal, the arcs intercepted by them are equal.		
Corollary: If two circles are congruent and if two chords of the circle are equal, the arcs intercepted by the chords are equal.		
Corollary: If two arcs of a circle are equal, the chords which they intercept are equal.		
c. If a diameter is perpendicular to a chord, it bisects the chord and the arcs determined by the chord.		
d. If a diameter bisects a chord which is not a diameter, it is perpendicular to the chord and bisects the arcs determined by the chord. (Problem 1, page 362)		
e. The perpendicular bisector of a chord contains the center of the circle.		
(This theorem should be treated informally.)		
f. Two chords of a circle are equal if and only if they are equidistant from the center of the circle.		
g. If two chords of a circle are unequal, their distances from the center of the circle are unequal in the opposite order.		

SCOPE	REFERENCES CODE	PAGES
h. If two chords of a circle are not equidistant from the center of the circle, the chords are unequal in the opposite order.		
2. Theorems pertaining to tangents		
a. A line is tangent to a circle if and only if it is perpendicular to a radius of the circle at its outer end. (This theorem should be treated informally.)		
b. The tangents to a circle from a point outside the circle make equal angles with the line joining the point and the center of the circle.	HPR...	378
c. The segments of tangents (tangent-segments) from a point outside the circle to the points of tangency are equal.		
I. Angles formed by chords, tangents, and secants (In the Appendix to the outline, there are three worksheets which will serve as excellent developmental exercises for the following theorems. The department chairman has been provided enough copies for all students.)		
1. Definitions		
a. Arc which an angle intercepts b. Arc in which an angle is inscribed		

SCOPE	REFERENCES	
	CODE	PAGES

2. Inscribed angle

   a. Definition: An angle whose vertex is a point of a circle and whose sides intersect the circle in two other different points is called an inscribed angle.

   b. Basic theorem: The measure of an inscribed angle is equal to one-half the degree measure of the arc it intercepts.

   Corollary: If an angle is inscribed in a semicircle, it is a right angle.

   Corollary: If two inscribed angles of a circle have equal arcs, the angles are equal.

   Corollary: If two inscribed angles of a circle are equal, the arcs intercepted by the angles are equal.

3. Chord-chord angle: The measure of the angle formed by two chords intersecting in the interior of the circle is one-half the sum of the measures of the intercepted arcs. (Problem 23, page 371)

4. Tangent-chord angle: The measure of the angle formed by a tangent and a chord is one-half the degree measure of the intercepted arc.

5. Secant-secant angle; tangent-secant angle; tangent-tangent angle: The measure of the angle formed by two secants, two tangents, or a tangent and a secant intersecting outside the circle is one-half the difference of the degree measures of the intercepted arcs.

SCOPE	REFERENCES	
	CODE	PAGES
J. Proportions involving chords, secants, and tangents	SMSG..	862-864
(These theorems are not in the text but they should be proved.)		
1. The product of the measures of a secant-segment from a given exterior point and its external secant-segment is constant for any secant containing the given point.		
2. The square of the measure of a tangent-segment from a given external point is equal to the product of the measures of a secant-segment and its external secant-segment from the given point.		
3. If two chords of a circle intersect, the product of the measures of the segments of one is equal to the product of the measures of the segments of the other.		
K. Inscribed and circumscribed polygons		
1. Definitions (review)	HPR...	382
2. Basic theorems	HPR...	384-385
(The theorems on concurrent lines should be reviewed before introducing these theorems.)		
a. A circle can be circumscribed about any triangle. b. A circle can be inscribed in any triangle. c. There exists one and only one circle through three points not in the same line.		

SCOPE	REFERENCES	
	CODE	PAGES
3. Theorem: A circle can be circumscribed about each regular polygon. Corollary: A circle can be inscribed in each regular polygon.	HPR	386
L. The linear measure of an arc 1. Definition: The circumference of a circle is its length expressed in terms of a linear unit. 2. Postulate: The ratio of the circumference of a circle to the length of a diameter of that circle is the same for every circle. This constant ratio is called pi (π). (A concise historical treatment of pi is given on page 496 of JDD.) 3. Basic formulas a. The circumference of a circle is the value of the expression πd. b. The measure of an arc $\overset{\frown}{AB}$ is the value of the expression $\pi d \, \frac{m(\overset{\frown}{AB})}{360}$. $\left(\frac{\text{Length of an arc}}{\text{Circumference}} = \frac{\text{Degree measure of the central angle}}{360} \right)$	HPR	389
II. Spheres (These concepts should be developed informally.)	HPR	443

SCOPE	REFERENCES	
	CODE	PAGES
A. Basic definitions		
1. Sphere		
2. Radius		
3. Congruent spheres		
B. Separation: A sphere separates space into three distinct subsets.		
1. The points of the sphere		
2. The points in the interior of the sphere: inside of the sphere		
3. The points in the exterior of the sphere: outside of the sphere		
C. Spheres in a coordinate system		
1. Equation of a sphere with center at the origin: $x^2 + y^2 + z^2 = r^2$	SMSG. .	812
2. Equation of a sphere with center at (a, b, c): $(x-a)^2 + (y-b)^2 + (z-c)^2 = r^2$ (This is called the general equation of the sphere.)		
D. Lines, segments, and planes related to spheres	HPR. . .	444
1. Chord		
2. Diameter		
3. Secant line; secant plane		
4. Tangent line; tangent plane		
5. Point of tangency		
E. Relationships of two spheres; lines and planes related to those spheres		
1. Concentric spheres		

SCOPE	REFERENCES	
	CODE	PAGES
2. Intersecting spheres		
3. Tangent spheres		
a. Internally tangent spheres		
b. Externally tangent spheres		
4. Tangents to two spheres		
a. Common tangent lines (planes)		
b. Common internal tangent		
c. Common external tangent		
F. Theorems relating to planes, secants and tangents of spheres (These theorems should be treated informally.)	HPR...	445-447
1. If a plane intersects a sphere in more than one point, the intersection is a circle.		
2. If a line is tangent to a sphere, it is tangent to the circle on the sphere which is the intersection of the sphere and a plane containing the line and the center of the sphere.		
3. If a line is tangent to a sphere, it is perpendicular to the radius of the sphere at the point of tangency.		
4. If a plane is tangent to a sphere, it is perpendicular to the radius of the sphere at the point of tangency.		

SCOPE	REFERENCES	
	CODE	PAGES
5. If a plane is perpendicular to a radius of a sphere at its endpoint on the sphere, it is tangent to the sphere.		
G. Circles on a sphere (The practical applications of these concepts can be illustrated by referring to the earth as a physical model of a sphere; the slate globe is an excellent visual aid to use in illustrating this analogy.)	HPR... MZ....	449 362-363 402-403
1. Definitions		
a. Great circle b. Small circle c. Axis d. Poles		
2. Theorem: The axis of a circle of a sphere contains the center of the circle.	HPR...	450
H. Distance on a sphere (This topic should be deleted for slower classes.)		
1. Definitions		
a. Spherical distance	HPR...	452
b. Polar distance	HPR...	455
c. Quadrant		

SCOPE	REFERENCES	
	CODE	PAGES
2. Theorem: All points of a circle of a sphere have the same spherical distance from a pole on the circle.	HPR...	454
Corollary: The degree measure of an arc determined by each point of a great circle and each pole of the circle is 90.		
I. Spherical angles	HPR...	456-457
(This topic should be deleted for slower classes.)		
1. Definitions		
a. Spherical angle		
(1) Vertex (2) Side		
b. The measure of a spherical angle		
2. Theorem: The degree measure of a spherical angle is equal to the degree measure of the arc of the great circle whose pole is the vertex of the angle and which is determined by the sides of the angle.		
J. Inscribed and circumscribed polyhedrons	HPR...	458
(This section should be done intuitively.)		

SCOPE	REFERENCES	
	CODE	PAGES
UNIT TEN AREAS OF PLANE FIGURES (Time allotment - 8 days)		
The basic text uses the terms "area" and "volume" as undefined terms. The teacher may wish to define these terms and present a more precise treatment of area and volume, such as the one found in Chapter 11 of SMSG.		
I. Basic concepts	JDD...	471-501
A. Physical concept of area; unit of area; area of a plane figure		
B. Postulate: The area of every geometric figure is the sum of the areas of the nonoverlapping figures of which it is composed.		
II. Polygons		
A. Postulates relating to polygons		
1. The area of a rectangle is the product of the lengths of two adjacent sides. (A = bh)		
2. If two polygons are congruent, they have the same area.		
B. Triangles	SMSG..	745-756
1. Review of definitions		

SCOPE

REFERENCES

CODE | PAGES

2. Theorems

   a. The area of a right triangle is one-half the product of the measures of the legs of the right triangle. $(A = \frac{1}{2}ab)$

   b. The area of every triangle is equal to one-half the product of the measures of a side and the altitude to that side. $(A = \frac{1}{2}bh)$

   c. The area of every triangle equals $\sqrt{s(s-a)(s-b)(s-c)}$, where a, b, and c represent the lengths of the respective sides of the triangle and $s = \frac{1}{2}(a + b + c)$.

   d. The area of an equilateral triangle with side s is one-fourth the product of the side squared and $\sqrt{3}$. $(A = \frac{s^2}{4}\sqrt{3})$

C. Quadrilaterals

1. Review of definitions

2. Theorems

   a. The area of a parallelogram is equal to the product of the measures of a side and an altitude to that side. $(A = bh)$

   b. The area of a trapezoid is equal to one-half the product of its altitude and the sum of the measures of its bases. $(A = \frac{1}{2}h[b + b'])$

   Corollary: The area of a trapezoid is equal to the product of its altitude and the measure of its median. $(A = hm)$

SCOPE	REFERENCES	
	CODE	PAGES
c. The area of a quadrilateral whose diagonals are perpendicular is equal to one-half the product of the diagonals. ($A = \frac{1}{2}dd'$)		
Corollary: The area of a rhombus is equal to one-half the product of the diagonals.		
D. Ratios of areas		
(Daily drills should contain many numerical applications of the following ratios.)		
1. Given a set of two or more triangles:	SMSG.	. 759
a. If the bases of all the triangles are equal, then the areas are proportional to the corresponding altitudes.		
b. If the altitudes of all the triangles are equal, then the areas of the triangles are proportional to the corresponding bases.		
c. If the areas of all the triangles are equal, then the bases of the triangles are inversely proportional to the corresponding altitudes.		
2. Given a set of two or more parallelograms:	SMSG.	. 761
a. If the bases of all the parallelograms are equal, then the areas of the parallelograms are proportional to the corresponding altitudes.		
b. If the altitudes of all the parallelograms are equal, then the areas of the parallelograms are proportional to the corresponding bases.		

SCOPE	REFERENCES	
	CODE	PAGES
c. If the areas of all the parallelograms are equal, then the bases of the parallelograms are inversely proportional to the corresponding altitudes.		
3. If two triangles are similar then the ratio of their areas is equal to the ratio of the squares of any two corresponding sides.	SMSG	765
E. Regular polygons		
1. Definitions	SMSG	771
a. Center b. Central triangle c. Radius d. Apothem		
2. Basic theorems		
a. The bisectors of the interior angles of a regular convex polygon of n sides intersect in a point.		
b. Every central triangle of a regular polygon is isosceles and is congruent to every other central triangle.		
3. Theorem: The area of a regular polygon is one-half the product of the apothem and the perimeter of the polygon. ($A = \frac{1}{2} ap$)		

SCOPE	REFERENCES CODE	 PAGES
III. Circles		
(The basic text postulates the formula for the area of a circle. It is advisable for the teacher to present an informal study of limits and to treat this formula as a theorem.)		
A. Definitions		
1. Circular region: A circular region is the union of a circle and its interior.		
2. Area of a circle: The area of a circle is the limit of the areas of the inscribed regular polygons.		
B. Theorem: The area of a circle of radius $\underline{r}$ is πr^2.		
C. Corollary: The areas of two circles are proportional to the squares of their radii.		
D. Sector of a circle	JDD	...500
1. Definition: A sector of a circle is the region bounded by two radii and an arc of the circle.		
2. Theorem: The area of a sector is one-half the product of the radius and the length of the arc.		
E. Segment of a circle	JDD	...501
1. Definition: A segment of a circle is the region bounded by a chord and an arc of the circle.		

SCOPE	REFERENCES	
	CODE	PAGES
2. Theorem: The area of a segment is the difference between the areas of the corresponding sector and the corresponding triangle formed by the two radii and the chord of the segment.		
(The teacher should stress that as the number of sides of the inscribed regular polygon increases, the following changes take place:	JDD. . .	494
1. The apothems increase and approach the length of the radius as a limit.		
2. The perimeters increase and approach the circumference of the circle as a limit.		
3. The areas increase and approach the area of the circle as a limit.)		

SCOPE	REFERENCES CODE | PAGES

UNIT ELEVEN

AREAS AND VOLUMES OF SOLID FIGURES

(Time allotment - 6 days)

The outline stresses the simultaneous development of prisms and cylinders and of pyramids and cones. The concept of a circle as the limit of a polygon of n sides has been introduced in Unit Nine. In this unit the concept of limit is extended to include:

1. The cylinder as the limit of an inscribed prism of n faces
2. The cone as the limit of an inscribed pyramid of n faces.

I. Prisms and cylinders

A. Basic concepts and definitions

SMSG. . 735-803
JDD. . . 517-549

1. Polygonal surface
2. Prismatic surface, cylindrical surface
3. Lateral element
4. Bases
5. Prism, cylinder
6. Edge
7. Lateral edge, lateral face
8. Vertex
9. Right section
10. Right prism, right circular cylinder
11. Parallelepiped, rectangular solid, cube
12. Altitude
13. Truncated prism

SCOPE	REFERENCES	
	CODE	PAGES

14. Prism inscribed in a circular cylinder
15. Circular cylinder inscribed in a prism

B. Postulate: All the properties of prisms whose bases are enclosed by regular polygons, and which do not depend upon the number of faces of the prism, are true also of circular cylinders.

C. Locus theorem: The locus of points in space at a given distance d from a given line l is a circular cylindrical surface with radius r.

D. Preliminary theorems

1. If two parallel planes intersect the parallel lines in a closed prismatic surface, the sections formed are congruent.

2. The opposite faces of a parallelepiped are congruent and parallel.

3. The diagonals of a rectangular solid are equal. (optional)

4. The diagonals of a rectangular solid bisect each other. (optional)

(If students have studied coordinates in three dimensions, they should set up the coordinates for the vertices of a rectangular solid and use coordinates to prove the two optional theorems.)

E. Area

1. Definitions

SCOPE	REFERENCES	
	CODE	PAGES
a. Lateral area b. Total area		
2. Theorems		
a. The lateral area of a prism is equal to the product of a lateral edge and the perimeter of a right section. ($LA = PC$)		
b. The lateral area of a circular cylinder is equal to the product of the circumference of a right section of the cylinder and an element. ($LA = C e$)		
c. The total area of a prism is equal to the sum of the lateral area and twice the area of one of the bases. ($TA = LA + 2B$)		
d. The total area of a circular cylinder is equal to the lateral area and the sum of twice the area of one of the bases. ($TA = LA + 2B$)		
F. Volume		
1. Basic concepts		
a. The unit of volume b. The volume of a solid c. Equal solids		
2. Postulates	HPR...	430-432
a. The volume of a right rectangular prism is the product of the length and width of the base and the length of an altitude of the prism. ($V = lwh$ or $V = Bh$)		

SCOPE	REFERENCES	
	CODE	PAGES
b. The volume of every solid is the sum of the volumes of the nonoverlapping solids of which it is composed.		
c. Cavalieri's Theorem: Given two solids and a plane. If, for every plane which intersects the solids and is parallel to the given plane, the two intersections have equal areas, then the two solids have equal volumes. (This is a restatement of Postulate 10/6 found on page 432 of the basic text.)	SMSG.	927
3. Theorems		
a. The volume of any prism is equal to the product of the area of its base and its altitude. (V = Bh)		
b. The volume of a circular cylinder is equal to the product of the area of its base and its altitude. (V = Bh)		
II. Pryamids and cones		
A. Basic concepts and definitions		
1. Pyramidal surface, conical surface 2. Lateral element 3. Nappes upper and lower 4. Vertex 5. Pyramid, cone 6. Axis, altitude, slant height 7. Base		

SCOPE	REFERENCES CODE	PAGES

8. Regular pyramid, right circular cone
9. Frustum
10. Plane tangent to a circular cone
11. Pyramid inscribed in a circular cone
12. Circular cone inscribed in a pyramid

B. Postulate: All the properties of pyramids whose bases are enclosed by regular polygons, and which do not depend upon the number of faces of the pyramids, are true also of circular cones.

C. Theorems on area

1. The lateral area of a regular pyramind is equal to one-half the product of its slant height and the perimeter of its base. ($LA = \frac{1}{2} ps$)

2. The lateral area of a right circular cone is equal to one-half the product of its slant height and the circumference of its base. ($LA = \frac{1}{2} Cs$)

3. The lateral area of a frustum of a regular pyramid is equal to one-half the product of its slant height and the sum of the perimeters of its bases. ($LA = \frac{1}{2}s[\,p + p'\,]$)

4. The lateral area of a frustum of a right circular cone is equal to one-half the product of its slant height and the sum of the circumferences of its bases. ($LA = \frac{1}{2}s[\,C + C'\,]$)

SCOPE	REFERENCES	
	CODE	PAGES
D. Theorems on volume 1. The volume of a pyramid is equal to one-third the product of the area of its base and its altitude. ($V = \frac{1}{3} Bh$) 2. The volume of a circular cone is one-third the product of the area of the base and its altitude. ($A = \frac{1}{3} Bh$)	HPR...435	
III. Spheres A. Theorem: The area of a sphere is equal to the area of four great circles of that sphere. ($A - 4\pi r^2$) B. Theorem: The volume of a sphere is equal to one-third the product of its radius and its area. ($A = \frac{4}{3}\pi r^3$) (The following activity summarizes the total concept of measurement with respect to similar figures: 1. Any two corresponding segments of similar figures have the same ratio as any two other corresponding segments. 2. The lateral areas and total areas of similar solids have the same ratio as the squares of any two corresponding segments. 3. The volumes of two similar solids have the same ratio as the cubes of any two corresponding segments.)	HPR...460	

APPENDIX I

SPECIAL RIGHT TRIANGLES

Certain right triangles appear frequently in problems of the physical world such as engineering and in problems of related mathematics courses such as trigonometry. These special right triangles may be classified into sets of triangles, each set containing only triangles that are similar to one another. You should be able to recognize the special right triangles discussed in this unit, to understand the relationships that exist for each set, and to apply these relationships in the solution of problems.

I. The 3, 4, 5 right triangles.

Since $3^2 + 4^2 = 5^2$, the converse of the Pythagorean Theorem tells us that a triangle whose sides have measures 3, 4, 5 is a right triangle. In a similar manner, we can conclude that if any positive number k is used as a constant of proportionality, then a triangle whose sides measure 3k, 4k, 5k is a right triangle. For example, any triangle whose sides measure 6, 8, 10 or 150, 200, 250 is a member of this set of right triangles.

Example 1. Find the length of the hypotenuse of a right triangle if the length of the two legs of the triangle are 18 and 24.

You should note that the triangle is a 3, 4, 5 right triangle with 6 as the constant of proportionality since $18 = 6 \cdot 4$. Therefore, the length of the hypotenuse of the triangle is $6 \cdot 5$ or 30.

Example 2. Triangle ABC is a right triangle with $\angle$ C as the right angle. AC = 75 and BC = 45. Find the length of AB.

Since the length of AC is $15 \cdot 5$ and the length of BC is $15 \cdot 3$, the triangle is a 3, 4, 5 triangle with 15 as the constant of proportionality. Therefore, AB must equal $15 \cdot 4$ or 60.

II. The 5, 12, 13 right triangles.

Explain why a triangle whose sides have measures of 5, 12, 13 is a right angle. Explain why a triangle whose sides have measures of 5k, 12k, and 13k is a right triangle if k is some positive number. Explain why all these triangles are similar. Give other examples of three numbers which are the respective measures of the sides of triangles in this set of similar right triangles.

Example 1. Find the length of the hypotenuse of a right triangle if the lengths of the two legs of the triangle are 15 and 36.

You should note that the triangle is a 5, 12, 13 right triangle with 3 as the constant of proportionality since $15 = 3 \cdot 5$ and $36 = 3 \cdot 12$. Therefore, the length of the hypotenuse of the right triangle is $3 \cdot 13$ or 39.

Example 2. Triangle ABC is a right triangle with $\angle$ C as the right angle. If the hypotenuse of the triangle has a measure of 78 and one of the legs of the triangle has a measure of 72, find the measure of the other leg of the triangle.

Since $78 = 6 \cdot 13$ and $72 = 6 \cdot 12$, the triangle is a 5, 12, 13 triangle with 6 as the constant of proportionality. Therefore, the measure of the other leg is $6 \cdot 5$ or 30.

III. The 30, 60, 90 triangles.

Unlike the sets of triangles in (I) and (II), the triangles in this set are usually designated by measures of angles rather than lengths of sides. Suppose that in $\triangle ABC$, $m\angle C = 90$, $m\angle B = 60$, $m\angle A = 30$. We know by Theorem $5 \cdot 7$ on page 202 of your text, that the length of BC is one-half the length of AB.

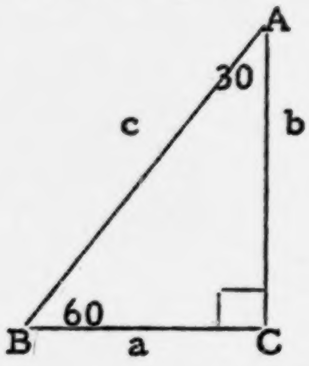
If we let $m(BC) = a$, then $m(AB) = c = 2a$, and by the Pythagorean Theorem

$$a^2 + b^2 = (2a)^2$$

$$a^2 + b^2 = 4a^2$$

$$b^2 = 3a^2$$

$$b = a\sqrt{3}$$



Thus we see that if $\triangle ABC$ is a 30, 60, 90 triangle $(a, b, c) \overset{=}{p} (1, \sqrt{3}, 2)$.

We can also prove the converse of this statement. If $\triangle A'B'C'$ is a triangle with $(B'C', C'A', A'B') \overset{=}{p} (1, \sqrt{3}, 2)$, then $\triangle A'B'C'$ is a 30, 60, 90 triangle.

We know that $\triangle A'B'C'$ is similar to $\triangle ABC$ by the SSS Similarity Theorem. Therefore, $m\angle A' = 30$, $m\angle B' = 60$, and $m\angle C' = 90$ because corresponding angles of similar triangles are equal in measure.

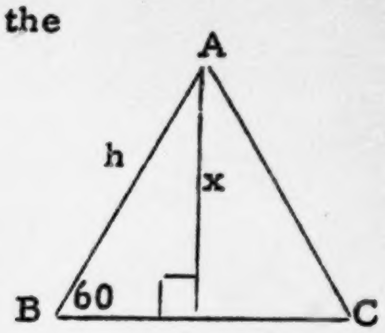
The above discussion can be summarized in the following theorem and corollaries:

Theorem A. Triangle ABC is a right triangle with $m\angle A = 30$, $m\angle B = 60$, $m\angle C = 90$ if and only if $(a, b, c) \overset{=}{p} (1, \sqrt{3}, 2)$.

Corollary 1. In a right triangle whose acute angles have measures of 30 and 60, the shorter leg is one-half the hypotenuse. $(s_{30} = \frac{h}{2})$

Corollary 2. In a right triangle whose acute angles have measures of 30 and 60, the longer leg is equal to one-half the hypotenuse multiplied by $\sqrt{3}$. $(s_{60} = \frac{h\sqrt{3}}{2})$

__Example 1.__ Find the length of the altitude of an equilateral triangle if the length of one of the sides is 8.



The altitude of an equilateral triangle bisects the vertex angle making a 30, 60, 90 triangle.

$(4, x, 8) \overset{=}{p} (1, \sqrt{3}, 2)$.

The constant of proportionality is 4.

Therefore, $x = 4\sqrt{3}$.

The problem can also be solved by using Corollary 2.

$$s_{60} = \frac{h\sqrt{3}}{2}$$

$$s_{60} = \frac{8 \cdot \sqrt{3}}{2}$$

$$s_{60} = 4\sqrt{3}.$$

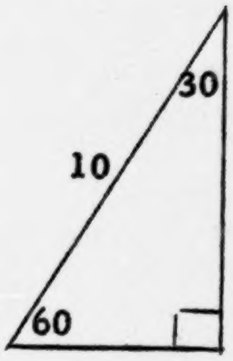
__Example 2.__ Find the measures of the sides of a 30, 60, 90 triangle if the length of the longer leg of the triangle is 10.

$(a, 10, c) \overset{=}{p} (1, \sqrt{3}, 2)$.

The constant of proportionality is $\frac{10}{\sqrt{3}}$.

Therefore, $a = \frac{10}{\sqrt{3}}$ and $c = \frac{2 \cdot 10,}{3}$ or

$$a = \frac{10\sqrt{3}}{3} \text{ and } c = \frac{20\sqrt{3}}{3}.$$



IV. __The 45, 45, 90 triangles.__

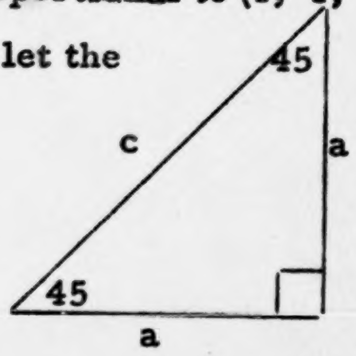
Since the triangles in this set have two angles that are equal in measure, they are sometimes called isosceles right triangles. We can prove that the lengths of the sides of any isosceles right triangle are proportional to $(1, 1, \sqrt{2})$.

In $\triangle ABC$. $m\angle C = 90$, $m\angle A = m\angle B = 45$. If we let the length of AC = a, then the length of BC = a. Why? By the Pythagorean Theorem

$$c^2 = a^2 + a^2$$

$$c^2 = 2a^2$$

$$c = a\sqrt{2}.$$



Therefore, we see that if $\triangle ABC$ is a 45, 45, 90 triangle, $(a, a, c) \overset{=}{p} (1, 1, \sqrt{2})$.

We shall now prove the converse of this statement. If $\triangle A'B'C'$ is a triangle with $(a', b', c') \overset{=}{p} (1, 1, \sqrt{2})$, then $\triangle A'B'C'$ is a 45, 45, 90 triangle.

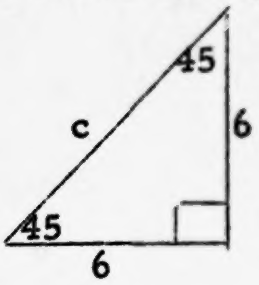
We know that $\triangle A'B'C'$ is similar to $\triangle ABC$ by SSS Similarity Theorem. Therefore, $m \angle C = 90$, $m \angle A = 45$, $m \angle B = 45$ because corresponding angles of similar triangles have equal measure.

The above discussion can be summarized in the following theorem and corollary.

Theorem B. Triangle ABC is a right triangle with $m \angle A = m \angle B = 45$ and $m \angle C = 90$ if and only if $(a, b, c) \overset{=}{p} (1, 1, \sqrt{2})$.

Corollary. In an isosceles right triangle, either leg of the triangle is equal to one-half the hypotenuse multiplied by $\sqrt{2}$. ($s_{45} = \frac{h\sqrt{2}}{2}$)

Example 1. Find the length of the hypotenuse of a right isosceles triangle if each leg of the triangle has a measure of 6.

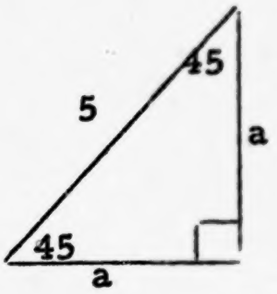


$(6, 6, c) \overset{=}{p} (1, 1, \sqrt{2})$

The constant of proportionality is 6.

Therefore, $c = 6\sqrt{2}$.

Example 2. Find the length of each leg of an isosceles right triangle if the length of the hypotenuse is 5.



$(a, a, 5) \overset{=}{p} (1, 1, \sqrt{2})$

The constant of proportionality is $\frac{5}{\sqrt{2}}$.

Therefore, $a = \frac{5}{\sqrt{2}}$, or $\frac{5\sqrt{2}}{2}$.

The problem can also be solved by using the corollary.

$$s_{45} = \frac{h\sqrt{2}}{2}$$

$$s_{45} = \frac{5\sqrt{2}}{2}.$$

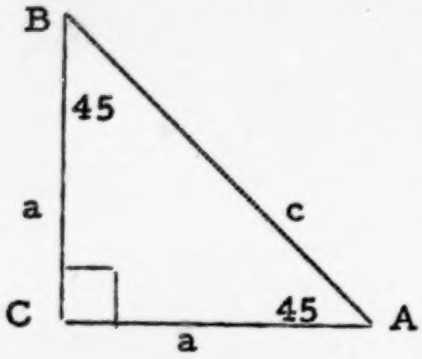
Problem Set

1. In each of the following exercises, the lengths of a leg and the hypotenuse of a right triangle are given. Which of the measures belong to a triangle similar to the 3, 4, 5 triangle? to the 5, 12, 13 triangle? to the 1, $\sqrt{3}$, 2 triangle? to the 1, 1, $\sqrt{2}$ triangle?

(a) 6, 10
(b) 12, 15
(c) 24, 25
(d) 15, 39
(e) 3, 6
(f) 3, $2\sqrt{3}$
(g) 8, 10
(h) 1.5, 2.5
(i) 6, $6 \cdot 5$
(j) 24, 26
(k) $3\sqrt{2}$, $5\sqrt{2}$
(l) $\sqrt{2}$, 2

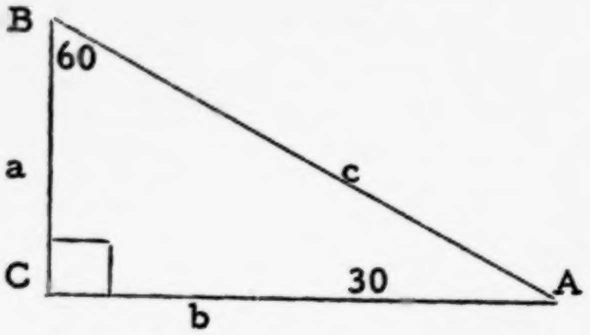
2. In each part of Problem 1, find the length of the side which is not given.

3. Complete the table for the right isosceles triangle in the diagram.



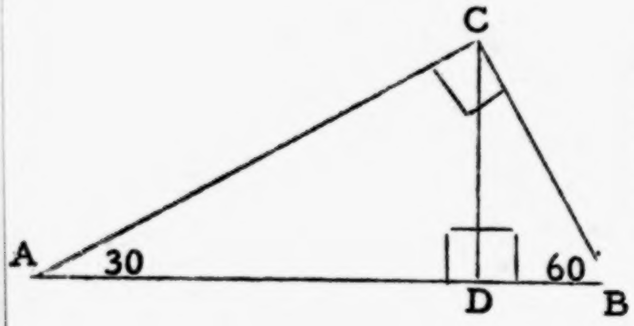
	a	c
(a)	10	
(b)	5	
(c)		9
(d)		6
(e)		$3\sqrt{2}$
(f)	$5\sqrt{2}$	

4. Complete the table for the 30-60-90 triangle in the diagram.



	a	b	c
(a)	10		
(b)	5		
(c)		18	
(d)		$9\sqrt{3}$	
(e)			20
(f)			$12\sqrt{3}$

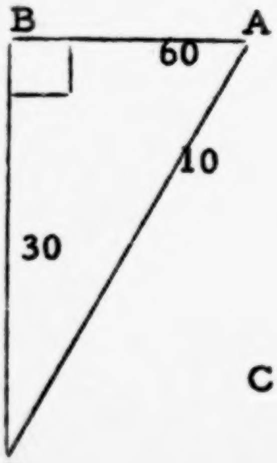
5. In each of the following exercises the length of one segment in the adjacent plane figure is given. Find the lengths of the remaining segments.



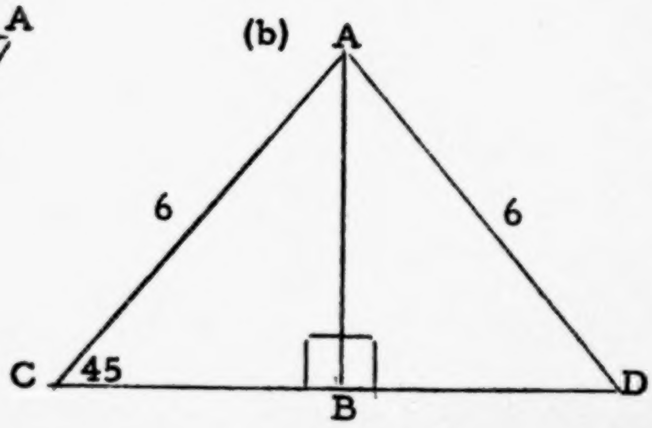
	m(AB)	m(BC)	m(CD)	m(AD)	m(DB)	m(AC)
(a)	8					
(b)		2				
(c)			$4\sqrt{3}$			
(d)				9		
(e)					$10\sqrt{3}$	
(f)						$8\sqrt{3}$

6. Use the three corollaries in this unit to find the length of AB and BC in each of the following plane figures. You should do all work mentally and write only the answer.

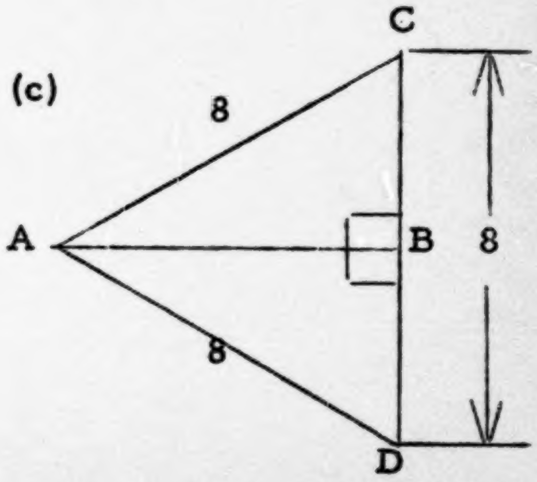
(a)



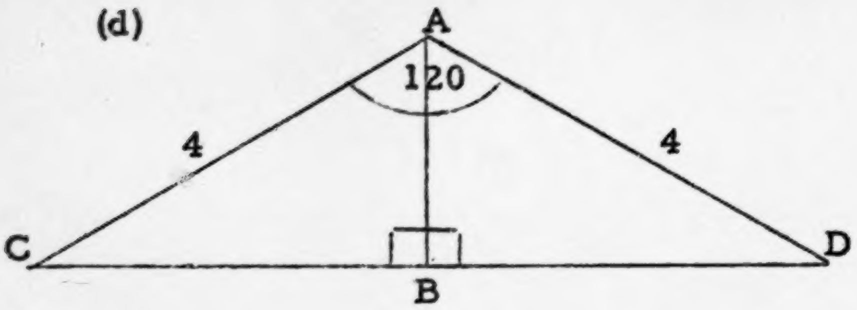
(b)



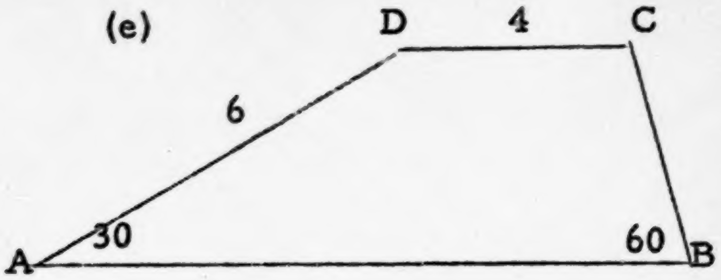
(c)



(d)

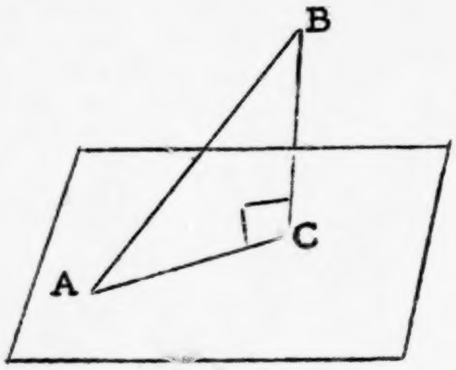


(e)



7. In the diagram, BC ⊥ AC and m∠A = 30.

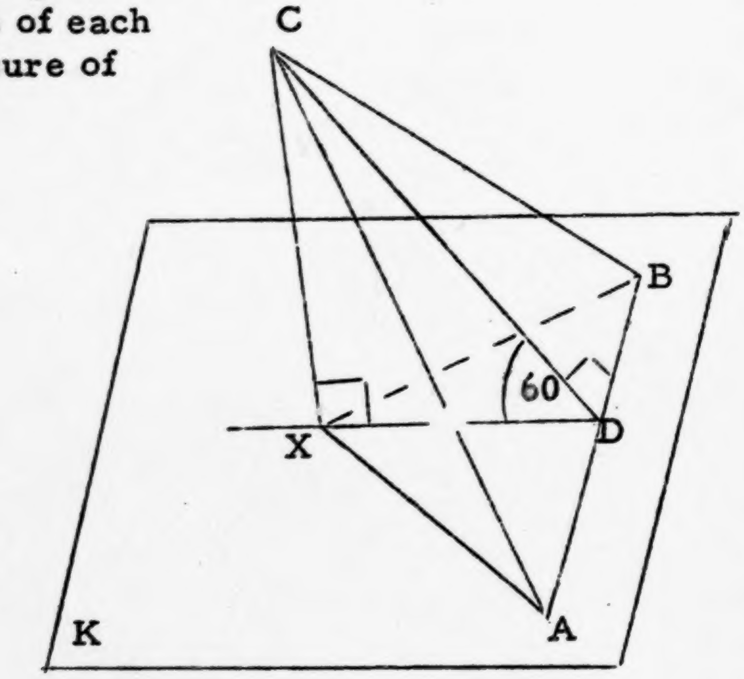
   (a) If m(AB) = 6, find m(AC).
   (b) If m(AB) = $4\sqrt{3}$, find m(AC).
   (c) If m(AC) = 9, find m(AB).
   (d) If m(AC) = $6\sqrt{3}$, find m(AB).



8. Repeat problem 7 if m∠A = 60.

9. In the diagram, △ABC is an equilateral triangle which is inclined at an angle to plane K. The measure of the dihedral angle C-AB-X is 60; CX is perpendicular to plane K; CD is perpendicular to AB. Find the measure of each of the following segments if the measure of AB = 6.

   (a) AC
   (b) BC
   (c) CD
   (d) CX
   (e) AX
   (f) BX



10. Repeat problem 9 if the measure of the dihedral angle is 30; (b) 45.

APPENDIX II

PROOFS USING COORDINATES

I. Slope

Theorem C-1a. Any two segments of a nonvertical line have the same slope.

Proof:

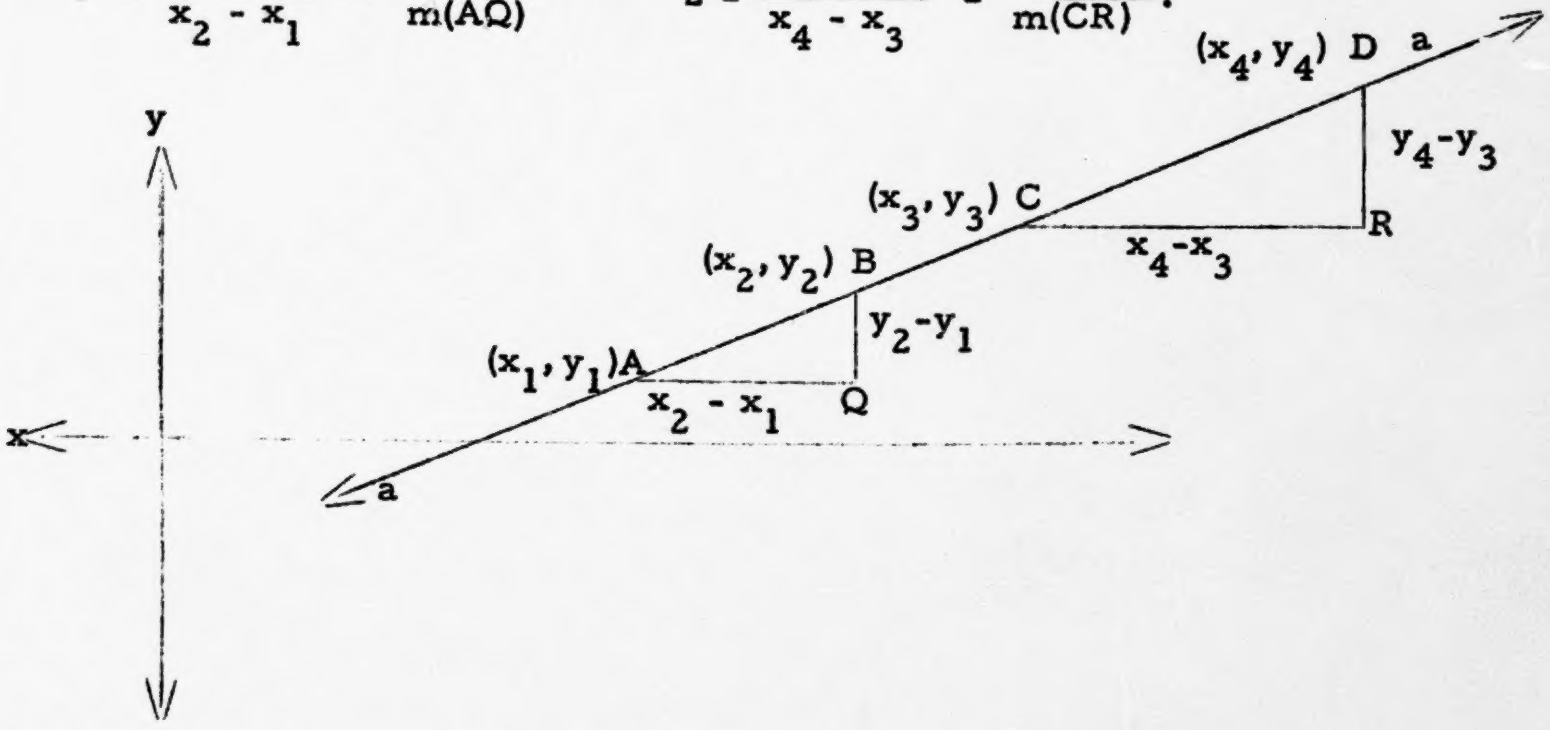
Line a is a nonvertical line. $A(x_1, y_1)$, $B(x_2, y_2)$, $C(x_3, y_3)$, and $D(x_4, y_4)$ are distinct points on a. Let m_1 be the slope of AB and m_2 be the slope of CD. We are required to prove that $m_1 = m_2$. The proof consists of two cases.

Case 1. If a is horizontal, then the y coordinates of all points on a are equal. Hence, $y_2 - y_1 = 0$ and $y_4 - y_3 = 0$. Since the x-coordinates of A, B, C, D are not equal, then $x_2 - x_1$ and $x_4 - x_3 \neq 0$.

Therefore, $\dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{y_4 - y_3}{x_4 - x_3} = 0$, and $m_1 = m_2$.

Case 2. If a is not a horizontal line, then

$$m_1 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{m(BQ)}{m(AQ)} \text{ and } m_2 = \frac{y_4 - y_3}{x_4 - x_3} = \frac{m(DR)}{m(CR)}.$$



∠BAQ = ∠DCR since equal corresponsind angles are formed when parallel lines are cut by a transversal.

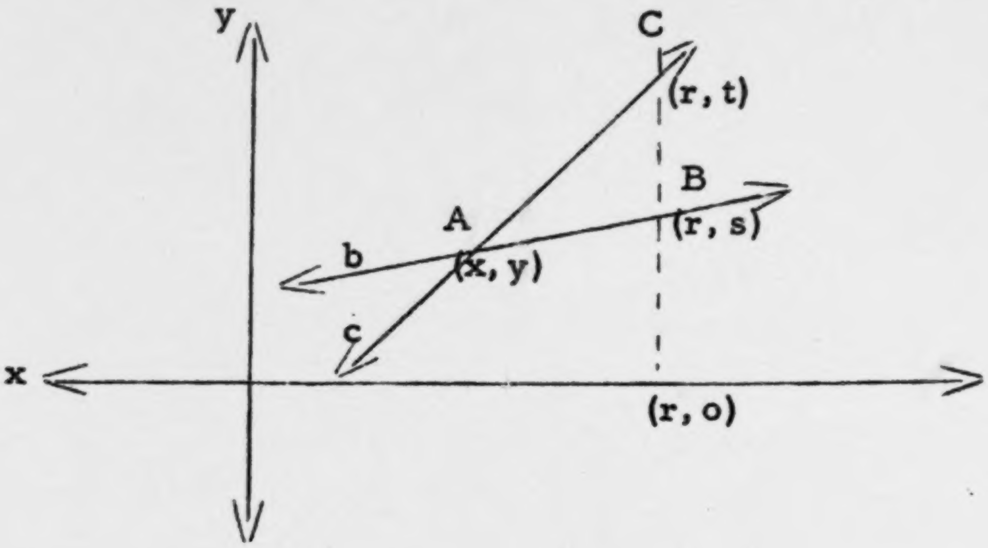
∠AQB = ∠CRD since right angles are equal.

ΔBAQ ∼ ΔCRD by the AA theorem for similarity.

$\dfrac{m(BQ)}{m(AQ)} = \dfrac{m(DR)}{m(CR)}$ since corresponding sides of similar triangles are proportional.

$m_1 = m_2$ by the substitution property for equality of numbers.

Theorem C-1b. Two distinct lines through the same point must have unequal slopes.



Let lines b and c intersect at point A (x, y).
Line b contains point B(r, s).
Since line c is distinct from line b, it cannot contain B. On line c at the point with x-coordinate equal to r, the y-coordinate must be some number t different from s. Then the slopes of b and c are unequal since $\frac{s-y}{r-x} \neq \frac{t-y}{r-x}$.

II. Parallel Lines

Theorem A-1. Two nonvertical lines are parallel if and only if they have equal slopes.

Proof:

Let a and b be the given lines and let their slopes be m_1 and m_2 respectively. There are two statements to prove.

(1) If $a \parallel b$, then $m_1 = m_2$.

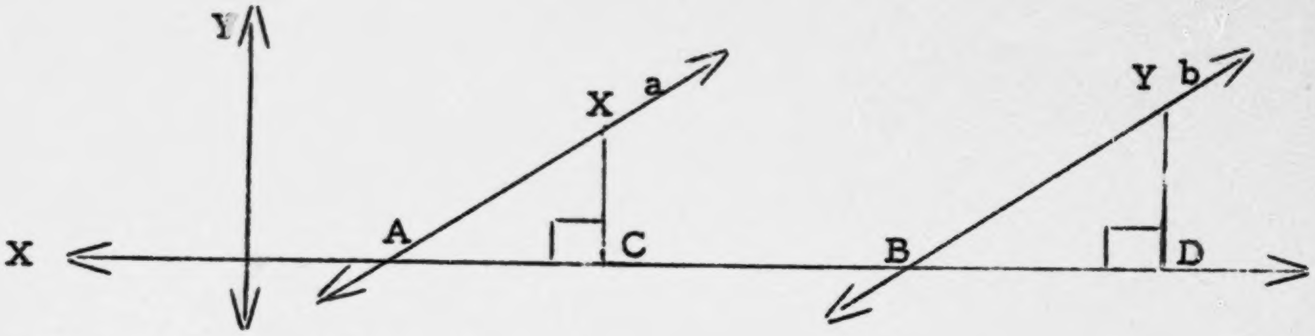
(2) If $m_1 = m_2$, then $a \parallel b$.

Part (1) consists of two cases.

Case 1. If a and b are parallel to the X-axis, they are horizontal lines and $m_1 = m_2 = 0$.

Case 2. If a and b are not horizontal, then they intersect the X-axis at points A and B, respectively. Perpendicular lines from point X on a and point Y on b intersect the X-axis at C and D, respectively.

Them $m_1 = \frac{M(\overline{CX})}{M(\overline{AC})}$ and $m_2 = \frac{M(\overline{DY})}{M(\overline{DB})}$.



$\angle XAC = \angle YBD$	since equal corresponding angles are formed when two parallel lines are cut by a transversal.
$\angle XCA = \angle YDB$	since right angles are equal.
$\Delta ACX \sim \Delta BDY$	by the AA theorem for similarity.
$\frac{M(\overline{CX})}{M(\overline{AC})} = \frac{M(\overline{DY})}{M(\overline{DB})}$	since corresponding sides of similar triangles are proportional.
$m_1 = m_2$	by the substitution property for equality of numbers.

Part (2).

If $m_1 = m_2$, then $a \parallel b$ since it is not possible for two distinct lines through the same point to have equal slopes. (Theorem 2)

III. Perpendicular Lines

Theorem B-1. Two nonvertical lines are perpendicular if and only if the product of their slopes is -1.

Proof:

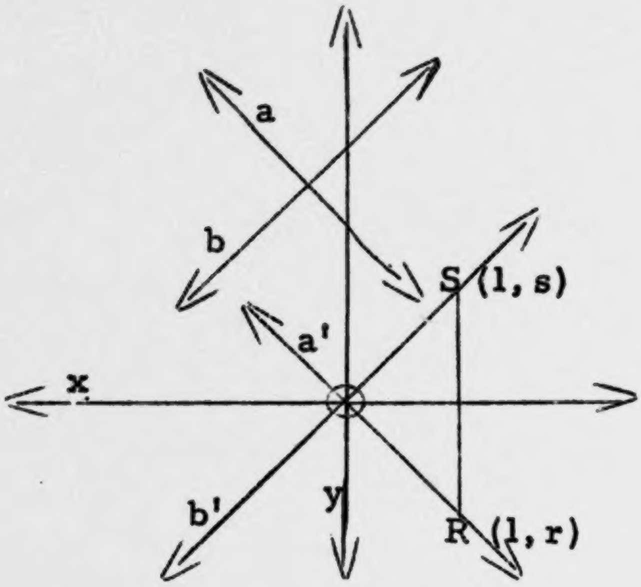
Let a and b be the given lines and let their slopes be m_1 and m_2 respectively. There are two statements to prove.

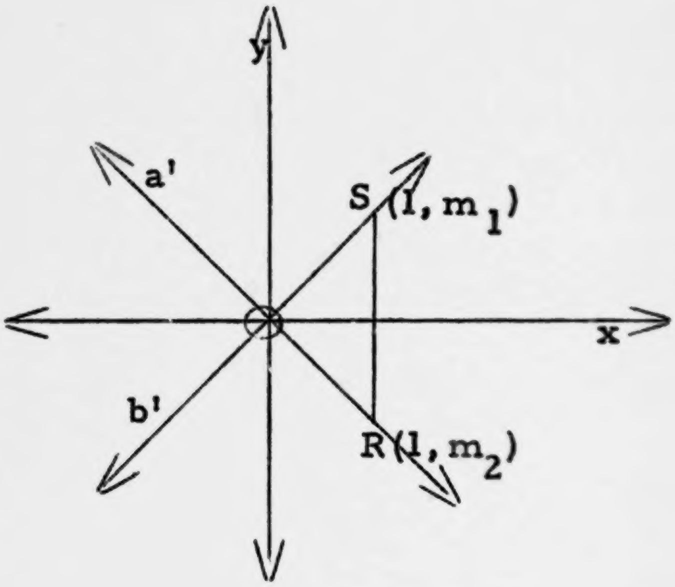
(1) If $a \perp b$, then $m_1 m_2 = -1$.

(2) If $m_1 m_2 = -1$, then $a \perp b$.

Both statements are proved together as follows.

Let a' be the line containing (0, 0) which is parallel to a, and let b' be the line containing (0, 0) which is parallel to b. The slope of a' = m_1 and the slope of b' = m_2. Let a' and b' intersect the vertical line x = 1 in R(1, r) and S(1, s), respectively.





$m_1 = \frac{r-0}{1-0} = r$, and $m_2 = \frac{s-0}{1-0} = s$.

Therefore, R = $(1, m_1)$ and S = $(1, m_2)$.

Then a $\perp$ b if and only if a' $\perp$ b' if and only if OR $\perp$ OS.

From the Pythagorean Theorem and its converse

a' $\perp$ b' if and only if $[m(OR)]^2 + [m(OS)]^2 = [m(RS)]^2$.

By the distance formula

$[m(OR)]^2 = 1 + m_1^2$; $[m(OS)]^2 = 1 + m_2^2$; $[m(RS)]^2 = (m_1 - m_2)^2$

Then a $\perp$ b if and only if $1 + m_1^2 + 1 + m_2^2 = m_1^2 - 2m_1m_2 + m_2^2$

$$2 = -2m_1m_2$$

$$m_1m_2 = -1$$

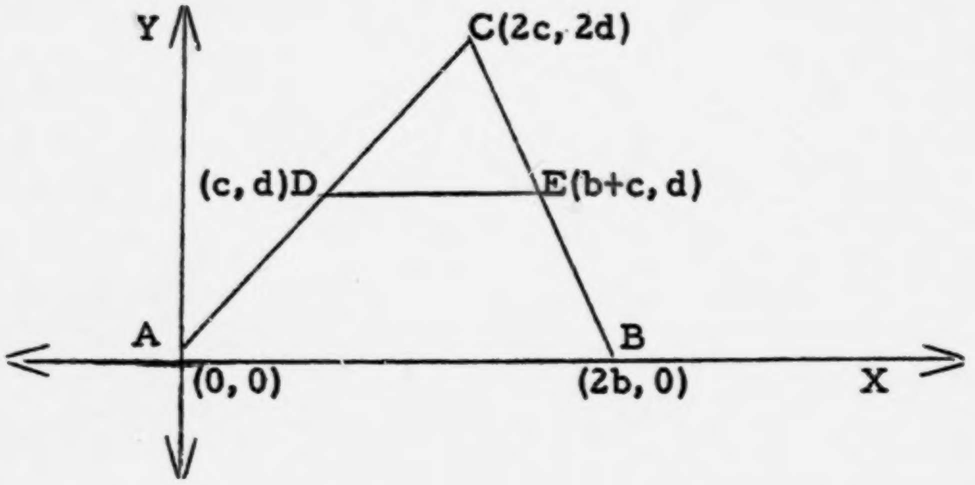
IV. Proof by Analytic Methods

Theorem C-1. The line segment determined by the midpoints of two sides of a triangle is parallel to the third, and its length is equal to one-half the length of the third side.

Proof:

Given $\triangle$ ABC with D and E as midpoints of AC and BD, respectively. Set up a coordinate system in the plane of the triangle with origin at point A and the positive part of the X-axis containing point B. Then A = (0, 0), B = (2b, 0), C = (2c, 2d), for some real numbers b, c, d. (b, c, d $\neq$ 0)

By the midpoint formula D = (c, d) and E = (b + c, d).



Since the y-coordinates of D and E are equal, then DE is parallel to the X-axis and hence parallel to AB.

By the distance formula: m(AB) = $| 2b - 0 | = | 2b |$

m(DE) = $| b + c - c | = | b |$

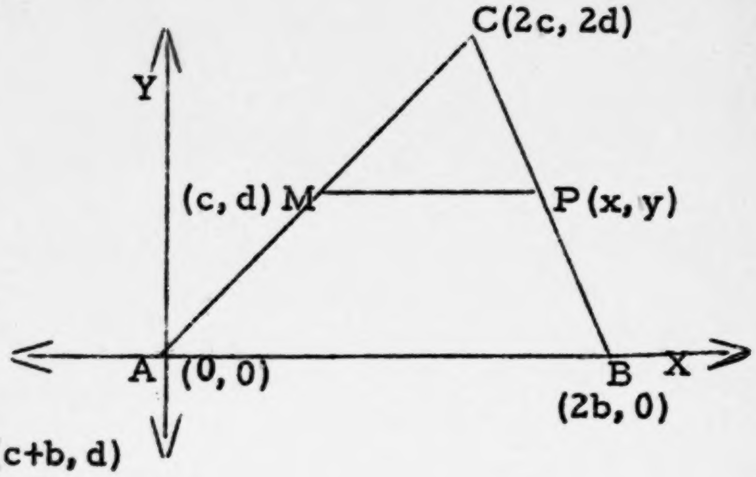
Therefore, m(DE) = $\frac{m(AB)}{2}$.

Theorem C-2. If a line bisects one side of a triangle and is parallel to a second side, it bisects the third side.

Proof:

Given $\triangle$ ABC with M as the midpoint of BC. A line through M parallel to AB intersects BC at P. Choose the same coordinate system as that chosen for Theorem C-1. A = (0, 0), B = (2b, 0), C = (2c, 2d). By the midpoint formula M = (c, d). Let P = (x, y).

Since MP is parallel to AB, it is a horizontal line and the y-coordinate of P is equal to the y-coordinate of M. Therefore, P = (x, d).



Since CB contains point P, the slope of CP must equal the slope of BP.

$$\frac{d}{2c - x} = \frac{d}{x - 2b}, \text{ and}$$

$$2c - x = x - 2b$$

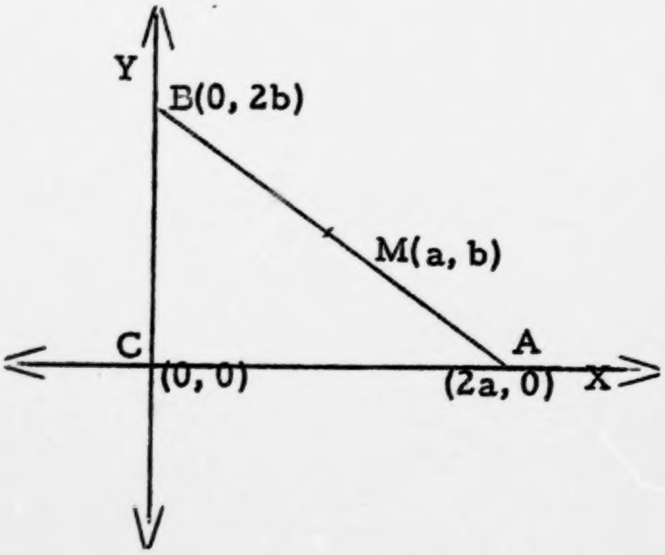
$$2c + 2b = 2x, \text{ and } x = c + d$$

Therefore, the coordinates of P are (c+b, d) which are the coordinates of the midpoint of segment BC. Hence MP bisects BC.

Theorem C-3. The midpoint of the hypotenuse of a right triangle is equidistant from all three vertices of the triangle.

Proof:

Given right triangle ABC with C as the vertex of the right angle. Let M be the midpoint of AB. Choose a coordinate system in the plane of ∠ABC with origin at point C and the positive part of the X-axis containing AC and the positive part of the Y-axis containing BC. Then C = 90, 0), A = (2a, 0), B = (0, 2b). By the midpoint formula M = (a, b).



By the distance formula

$$m(MC) = \sqrt{|a-0|^2 + |b-0|^2}$$
$$= \sqrt{a^2 + b^2}$$
$$m(MA) = \sqrt{|2a-a|^2 + |b-0|^2}$$
$$= \sqrt{a^2 + b^2}$$
$$m(MB) = \sqrt{|a-0|^2 + |2b-b|^2}$$
$$= \sqrt{a^2 + b^2}$$

Therefore, m(MB) = m(MA) = m(MC).

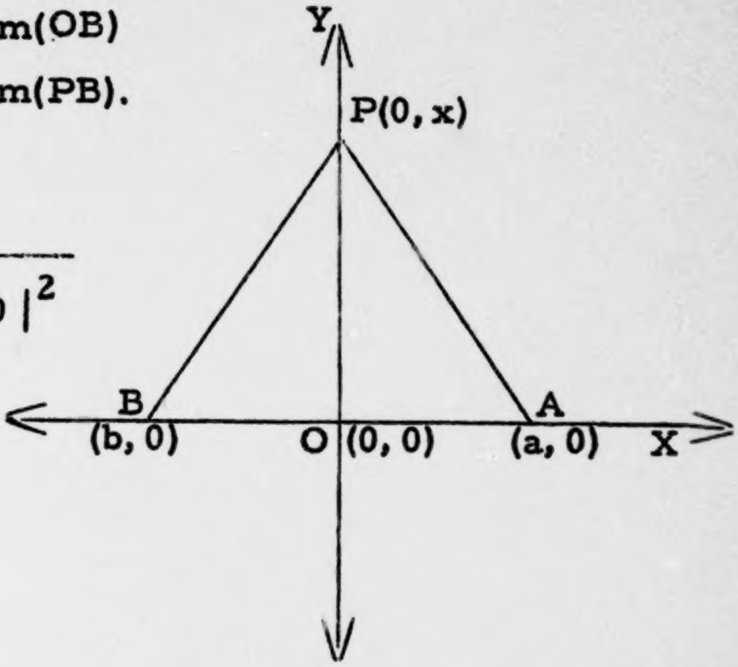
Theorem C-4. Oblique line segments from a point on a perpendicular to a line are equal in length if and only if they cut off segments of equal length from the foot of the perpendicular.

Proof:

Choose a coordinate system so that the axes are the given perpendicular lines. Let point P be contained in the Y-axis and points A and B be contained in the X-axis. Let P = (0, x), A = (a, 0), B = (b, 0). The intersection of the two perpendicular lines is the origin, 0 = (0, 0). There are two statements to prove.

(1) If m(PA) = m(PB), then m(OA)= m(OB)

(2) If m(OA) = m(OB), then m(PA) = m(PB).



(1) If m(PA) = m (PA) = m(PB), then

$$\begin{aligned} \sqrt{|a-0|^2 + |x-0|^2} &= \sqrt{|b-0|^2 + |x-0|^2} \\ a^2 + x^2 &= b^2 + x^2 \\ a^2 &= b^2 \\ a &= b \\ m(OA) &= m(OB) \end{aligned}$$

(2) If m(OA) = m(OB) then $|a| = |b|$.

Since A and B are distinct points b $\neq$ a. Therefore, b = -a, and the coordinates of B are (-a, 0). By the distance formula

$$m(PB) = \sqrt{|-a-0|^2 + |x-0|^2} = \sqrt{a^2 + x^2}.$$

$$m(PA) = \sqrt{|a-0|^2 + |x-0|^2} = \sqrt{a^2 + x^2}.$$

Therefore, m(PB) = m(PA).

Theorem C-5. If two oblique line segments from a point on a perpendicular to a line are unequal in length, then they cut off unequal segments from the foot of the perpendicular; the longest segment cuts off the greater distance from the foot of the perpendicular.

Proof:

Use the same coordinate system and diagram as the one used in Theorem C-4. It is given that m(PA) > m(PB), and we are to prove m(OA) > m (OB).

If m(PA) > m(PB), then

$$\sqrt{a^2 + x^2} > \sqrt{b^2 + x^2}$$

$$a^2 > b^2$$

$$|a| > |b|$$

$$m(OA) > m(OB).$$

Theorem C-6. The medians to the two equal sides of an isosceles triangle are equal in length.

Proof:

Given the isosceles triangle ABC with M, the midpoint of AC and N, the midpoint of AB. Choose a coordinate system in the plane of the triangle so that BC is contained in the X-axis and the Y-axis passes through A. Then A = (0, 2a), B = (2b, 0), C = (-2b, 0). By the midpoint formula M = (-b, a) and N = (b, a).

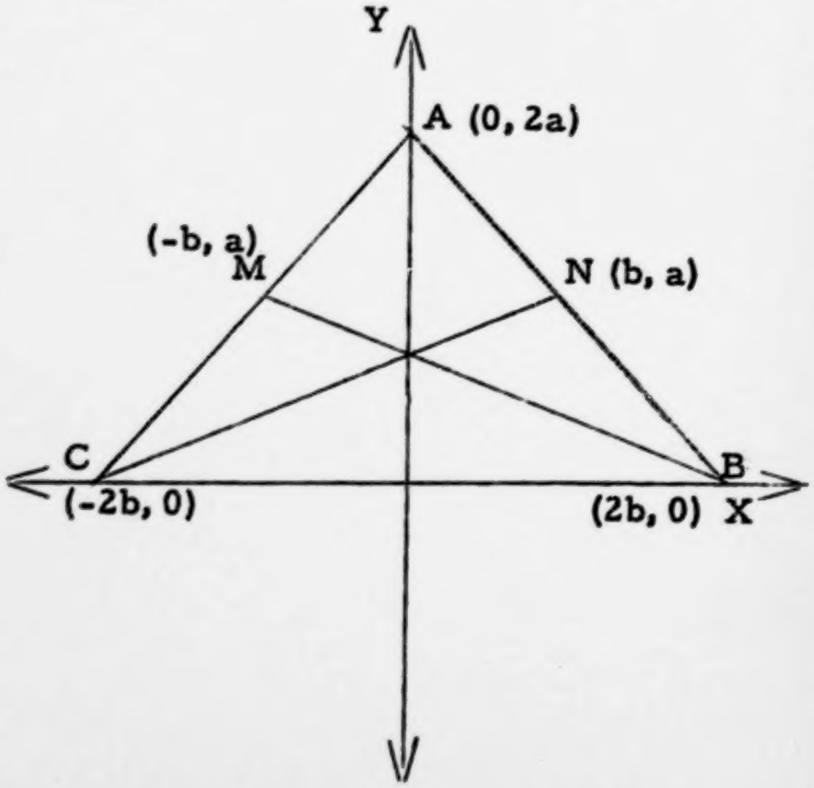
$$m(BM) = \sqrt{|2b + |^2 + |a-0|^2}$$

$$= \sqrt{9b^2 + a^2}$$

$$m(CN) = \sqrt{|-2b - b|^2 + |a-0|^2}$$

$$= \sqrt{9b^2 + a^2}$$

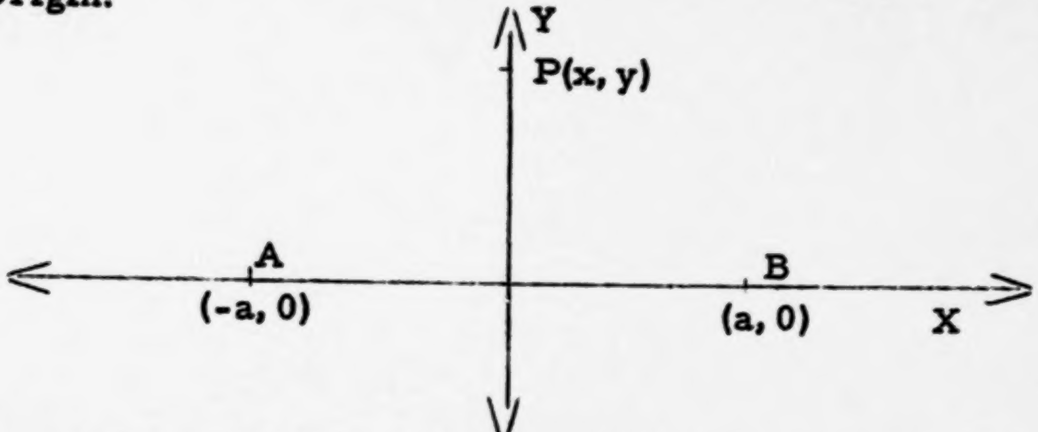
$$m(BM) = m(CN).$$



Theorem C-7. In a plane, a point is on the perpendicular-bisector of a line segment if and only if it is equidistant from the endpoints of the segment.

Proof:

Given two points A and B and a plane which contains them. Choose a coordinate system so that line AB is the X-axis and the midpoint of segment AB is the origin.



There is a real number a, $a \neq 0$, such that A = (-a, 0) and B = (a, 0). Then the y-axis is the perpendicular-bisector of segment AB. There are two parts to the proof.

(1) If P is in the Y-axis, then m(AP) = m(PB).

(2) If m(AP) = m(PB), and P is in the xy-plane, then P is in the Y-axis.

(1) If P is in the Y-axis, then P = (0, b) for some number b and

$$m(AP) = \sqrt{|-a-0|^2 + |0-b|^2} = \sqrt{a^2 + b^2}.$$

$$m(BP) = \sqrt{|a-0|^2 + |0-b|^2} = \sqrt{a^2 + b^2}.$$

Therefore, m(AP) = m(BP).

(2) If P(x, y) is any point such that m(AP) = m(BP), then by the distance formula

$$\sqrt{|x+a|^2 + |y-0|^2} = \sqrt{|x-a|^2 + |y-0|^2}$$

$$x^2 + 2ax + a^2 + y^2 = x^2 - 2ax + a^2 + y^2$$

$4ax = 0$, and since $a \neq 0$,

$x = 0$.

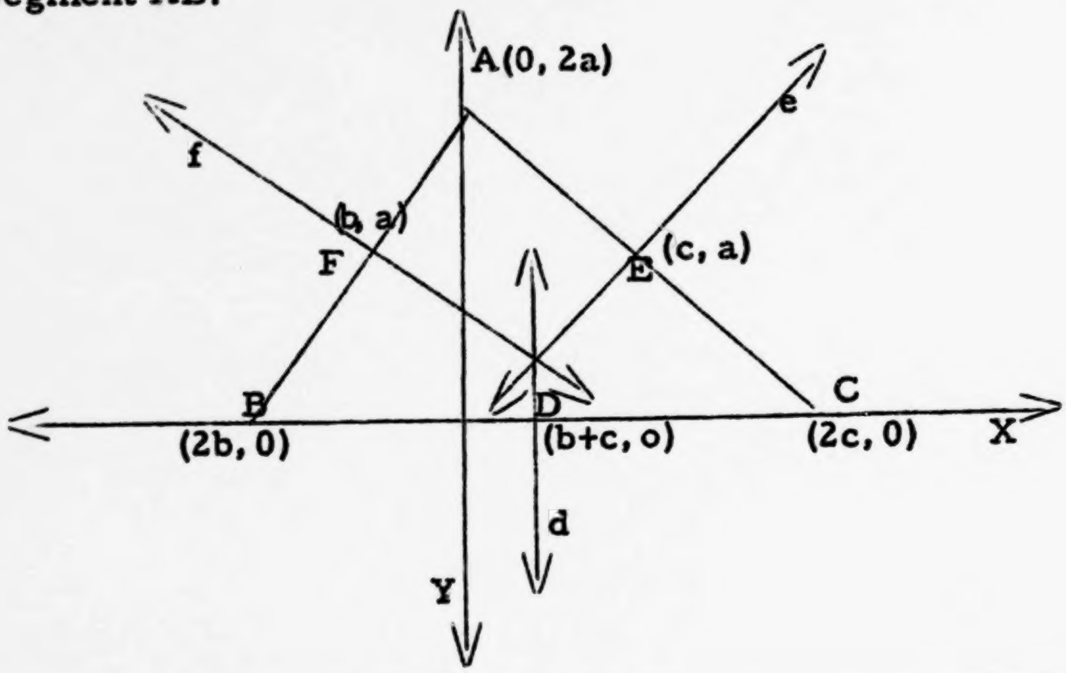
Therefore, P is in the Y-axis.

V. Concurrent Lines

Theorem D-2a. The perpendicular bisectors of the sides of a triangle are concurrent at a point equidistant from the vertices of the triangle.

Proof:

Given $\triangle ABC$ with D, E, F as the midpoints of segments BC, AC, AB respectively. Choose a coordinate system in the plane of the triangle such that the X-axis contains segment BC and the Y-axis contains point A. Then $A = (0, 2a)$, $B = (2b, 0)$, $C = (2c, 0)$. By the midpoint formula $D = (b+c, 0)$, $E = (c, a)$, $F = (b, a)$. Let d be the perpendicular-bisector of segment BC, e be the perpendicular-bisector of segment AC, and f the perpendicular-bisector of segment AB.



Segment		Perpendicular-bisector		
Name	Slope	Name	Slope	Equation
AB	$-\frac{a}{b}$	f	$\frac{b}{a}$	$y-a = \frac{b(x-b)}{a}$ (point-slope form)
AC	$-\frac{a}{c}$	e	$\frac{c}{a}$	$y-a = \frac{c(x-c)}{a}$ (point-slope form)
BC	0	d	not defined	$x = b+c$

By solving the equations for lines f and d simultaneously, we can find the point of intersection of the perpendicular-bisectors of segments AB and BC.

$$y - a = \frac{b(b+c-b)}{a}; \quad \text{and } y = \frac{bc}{a} + a.$$

Therefore, the coordinates of the point of intersection of d and e are $(b+c), \frac{bc}{a} + a)$.

By solving the equations for lines e and d simultaneously, we can find the point of intersection of the perpendicular-bisectors of segments AC and BC.

$$y-a = \frac{c(b+c-c)}{a}, \text{ and } y = \frac{bc}{a} + a.$$

Therefore, the coordinates of the point of intersection of c and f are $(b+c), \frac{bc}{a} + a)$.

Since the three perpendicular bisectors contain the point whose coordinates are $(b+c), \frac{bc}{a} + a)$, they are concurrent at that point.

Let the point of concurrency of the three perpendicular-bisectors be P. Then by the distance formula

$$m(AP) = \sqrt{|b+c|^2 + \left|\frac{bc}{a} - a\right|^2}$$

$$= \sqrt{b^2 + 2abc + c^2 + \frac{b^2c^2}{a^2} - 2bc + a^2}$$

$$= \sqrt{a^2 + b^2 + c^2 + \frac{b^2c^2}{a^2}}$$

$$m(BP) = \sqrt{|c-b|^2 + \left|\frac{bc}{a} + a\right|^2}$$

$$= \sqrt{c^2 - 2bc + b^2 + \frac{b^2c^2}{a^2} + 2bc + a^2}$$

$$= \sqrt{a^2 + b^2 + c^2 + \frac{b^2c^2}{a^2}}$$

$$m(CP) = \sqrt{|b-c|^2 + \left|\frac{bc}{a} + a\right|^2}$$

$$= \sqrt{b^2 - 2bc + c^2 + \frac{b^2c^2}{a^2} + 2bc + a^2}$$

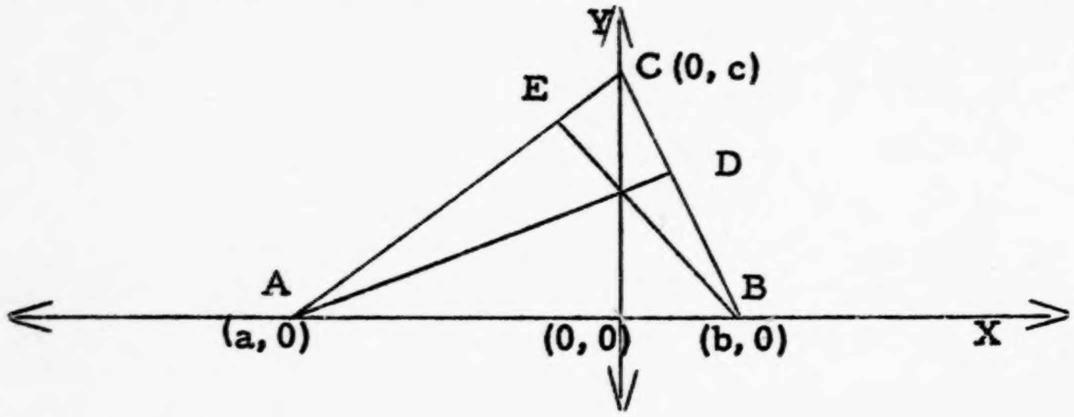
$$= \sqrt{a^2 + b^2 + c^2 + \frac{b^2c^2}{a^2}}$$

Therefore, m(AP) = m(BP) = m(CP).

Theorem D-2c. The lines which contain the altitudes of a triangle are concurrent.

Proof:

Given $\triangle ABC$ with altitudes AD, BE, and CF. Choose a coordinate system in the plane of $\triangle ABC$ so that the X-axis will contain segment AB and the Y-axis will contain segment CF. Then $A = (a, 0)$, $B = (b, 0)$, $C = (0, c)$.



1. Since the slope of segment $BC = -\frac{c}{b}$, the slope of the altitude $AD = \frac{b}{c}$, and the equation of line AD is $y = \frac{b(x-a)}{c}$.
2. Since the slope of segment $AC = -\frac{c}{a}$, the slope of the altitude $BE = \frac{a}{c}$, and the equation of line BE is $y = \frac{a(x-b)}{c}$.
3. Since the altitude CF is contained in the Y-axis, the equation of line CF is $x = 0$.

If we solve the equations for lines AD and CF simultaneously, we obtain $x = 0$ and $y = -\frac{ab}{c}$ and the coordinates for the point of intersection of these lines is $(0, -\frac{ab}{c})$.

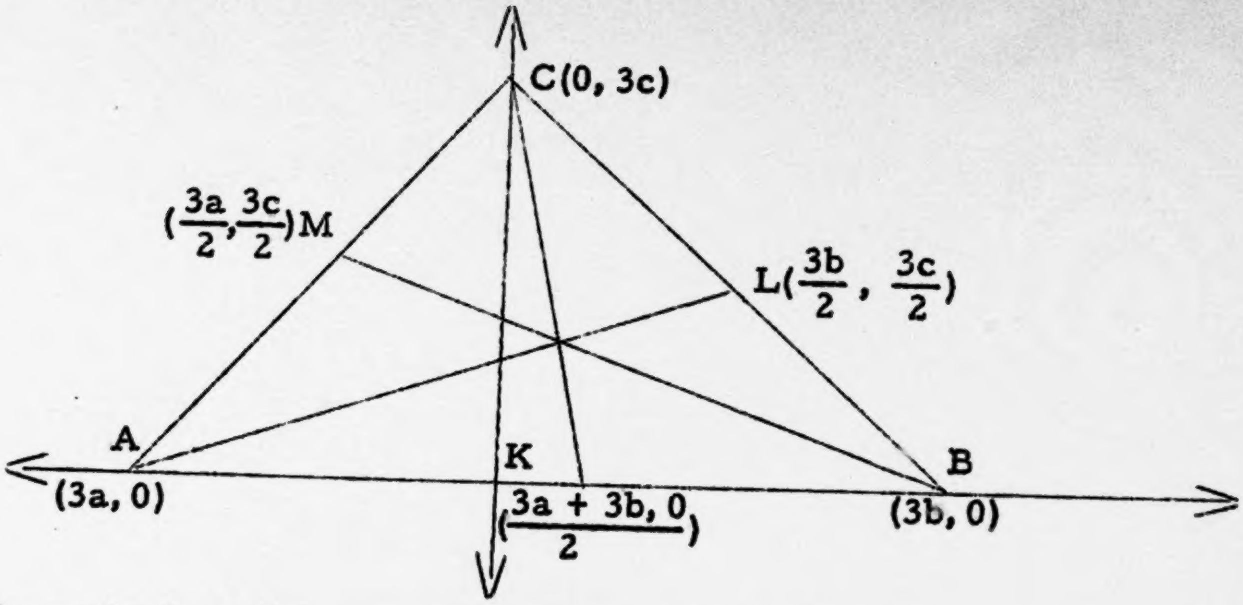
If we solve the equations for lines BE and CF simultaneously, we obtain $x = 0$ and $y = -\frac{ab}{c}$, and the coordinates for the point of intersection of these lines is $(0, -\frac{ab}{c})$.

Since $(0, -\frac{ab}{c})$ is contained in lines AD, BE, and CF, we conclude that they are concurrent at the points whose coordinates are $(0, -\frac{ab}{c})$.

Theorem D-2d. The medians of a triangle are concurrent at a point that divides each of the medians in the ratio of two to one.

Proof:

Given $\triangle ABC$ with K, L, M as the midpoints of segments AB, BC, AC respectively. Choose a coordinate system in the plane of the triangle such that the X-axis contains AB and the Y-axis contains point C. Let $A = (3a, 0)$, $B = (3b, 0)$, $C = (0, 3c)$. Then by the midpoint formula $K = (\frac{3a + 3b, 0}{2})$, $L = (\frac{3b}{2}, \frac{3c}{2})$, $M = (\frac{3a}{2}, \frac{3c}{2})$.


C(0, 3c)
$(\frac{3a}{2}, \frac{3c}{2})$M
L$(\frac{3b}{2}, \frac{3c}{2})$
A
(3a, 0)
K
$(\frac{3a + 3b}{2}, 0)$
B
(3b, 0)

By the ratio formula, we can find the coordinates of point P that divides AL into the ratio of 2:1, the coordinates of point P' that divides BM into the ratio of 2:1, and the coordinates of point P" that divides CK into the ratio of 2:1.

$P = (x, y)$

$$x = \frac{x_A + 2x_L}{3} \qquad y = \frac{y_A + 2y_L}{3}$$

$$x = \frac{3a + 2\frac{(3b)}{2}}{3} \qquad y = 0 + 2\,\frac{(3c)}{2}$$

$$x = \frac{3a + 3b}{3} \qquad y = \frac{3c}{3}$$

$$x = a + b \qquad y = c$$

$P = (a + b, c)$

$P' = (x', y')$

$$x' = \frac{x_B + 2x_M}{3} \qquad y' = \frac{y_B + 2y_M}{3}$$

$$x' = \frac{3b + 2\,\frac{(3a)}{2}}{3} \qquad y' = \frac{0 + 2\,(\frac{3c}{2}}{3}$$

$$x' = \frac{3a + 3b}{3} \qquad y' = \frac{3c}{3}$$

$$x' = a + b \qquad y' = c$$

$P' = (a + b, c)$

$$P'' = (x'', y''$$

$$x'' = \frac{x_C + 2x_K}{3} \qquad y'' = \frac{y_C + 2y_K}{3}$$

$$x'' = \frac{0 + 2\ \frac{(3a + 3b}{2}}{3} \qquad y'' = \frac{3c + 0}{3}$$

$$x'' = \frac{3a + 3b}{3} \qquad y'' = \frac{3c}{3}$$

$$x'' = a + b \qquad y'' = c.$$

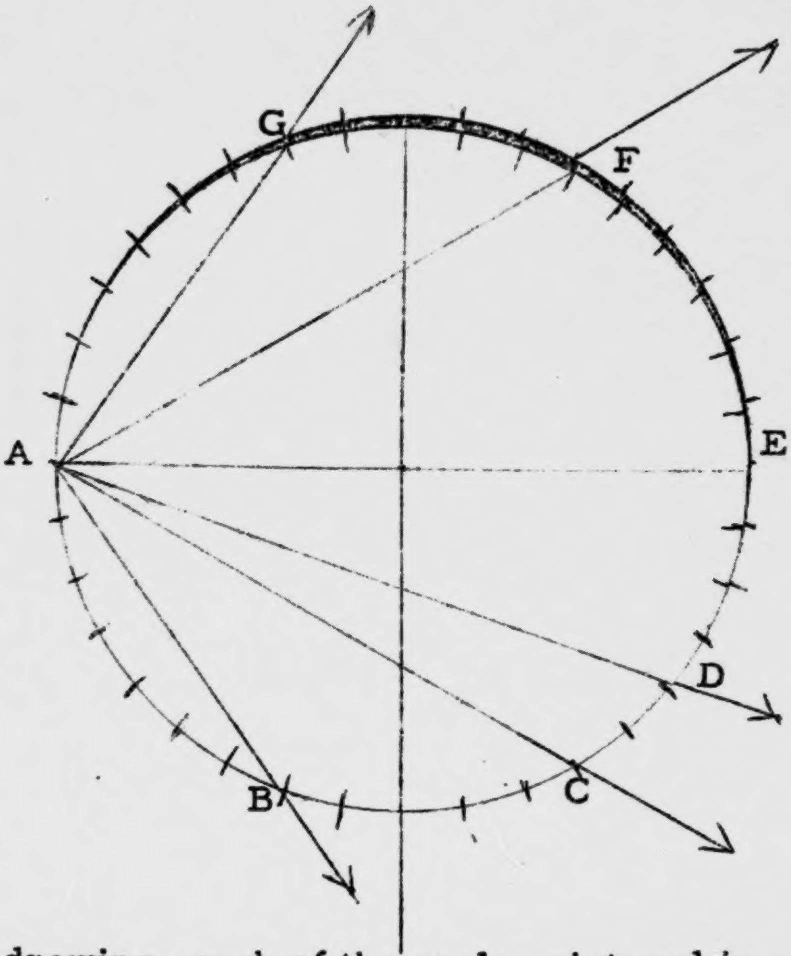
$$P'' = (a + b, c)$$

Therefore, $P = P' = P''$ and the medians are concurrent at a point whose coordinates are $(a + b, c)$.

APPENDIX III

EXPERIMENTS: MEASUREMENT OF ANGLES AND ARCS IN A CIRCLE

Experiment #1: Inscribed Angles



1. In the above drawing, each of the angles pictured is an inscribed angle. With your protractor, measure each of these angles to the nearest five degrees. Record the measurements in the chart on the following page.

2. Find the measure of each of the intercepted arcs by counting the number of degrees in each of them. (Each division of the circle represents an arc with a degree measure of ten.) Record these measurements in the chart.

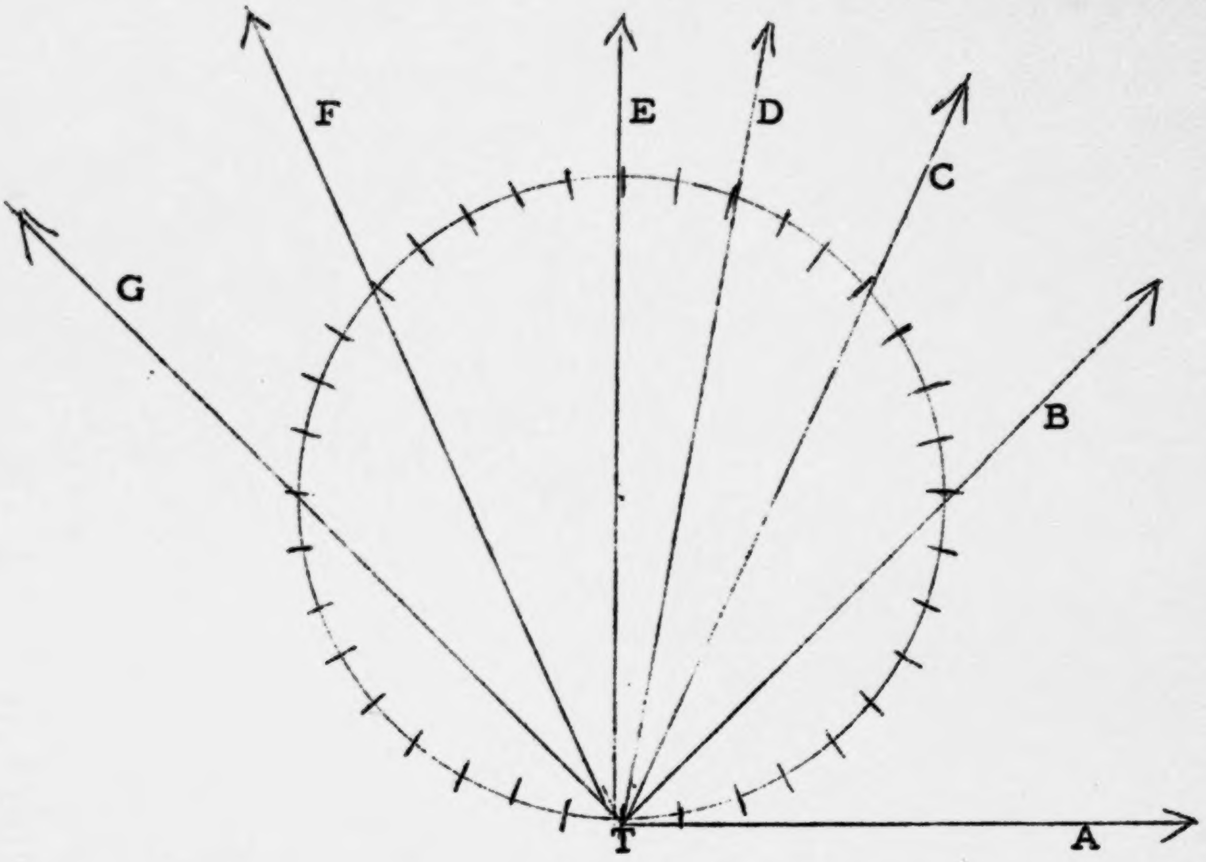
3. Try to discover a method of determining the measure of an inscribed angle if the measure of the intercepted arc is given.

4. What conclusion can you make concerning the measure of inscribed angles that intercept the same arc or arcs of equal measure?

5. What do you think is the measure of an angle that is inscribed in a semicircle?

Measure of Angle	Measure of Arc
a. $m\angle BAC$ = ______________	$m(\overset{\frown}{BC})$ = ______________
b. $m\angle BAD$ = ______________	$m(\overset{\frown}{BD})$ = ______________
c. $m\angle BAE$ = ______________	$m(\overset{\frown}{BE})$ = ______________
d. $m\angle BAF$ = ______________	$m(\overset{\frown}{BF})$ = ______________
e. $m\angle BAG$ = ______________	$m(\overset{\frown}{BG})$ = ______________
f. $m\angle CAD$ = ______________	$m(\overset{\frown}{CD})$ = ______________
g. $m\angle CAE$ = ______________	$m(\overset{\frown}{CE})$ = ______________
h. $m\angle CAF$ = ______________	$m(\overset{\frown}{CF})$ = ______________
i. $m\angle CAG$ = ______________	$m(\overset{\frown}{CG})$ = ______________
j. $m\angle DAE$ = ______________	$m(\overset{\frown}{DE})$ = ______________
k. $m\angle DAF$ = ______________	$m(\overset{\frown}{DF})$ = ______________
l. $m\angle DAG$ = ______________	$m(\overset{\frown}{DG})$ = ______________
m. $m\angle EAF$ = ______________	$m(\overset{\frown}{EF})$ = ______________
n. $m\angle EAG$ = ______________	$m(\overset{\frown}{EG})$ = ______________
o. $m\angle FAG$ = ______________	$m(\overset{\frown}{FG})$ = ______________

Experiment #2: Tangent and Chord Angles



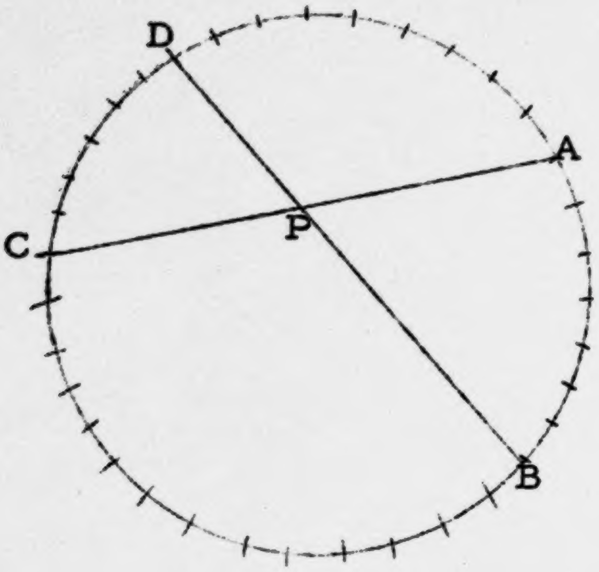
1. In the above drawing, the sides of each of the angles pictured are a tangent-ray and a chord - ray. With your protractor, measure each of these angles to the nearest five degrees. Record the measurements in the chart at the bottom of the page.

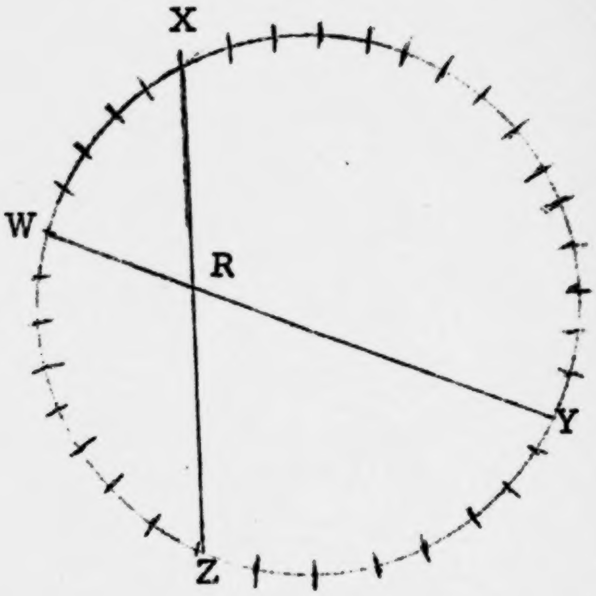
2. Find the measure of each of the intercepted arcs by counting the number of degrees in each of them. (Each division of the circle represents an arc with a degree measure of ten.) Record these measurements in the chart.

3. Try to discover a method of determining the measure of a tangent - chord angle if the measure of the intercepted arc is given.

Measure of Angle	Measure of Arc
a. m∠ ATB = ______	$m(\widehat{TB})$ = ______
b. m∠ ATC = ______	$m(\widehat{TC})$ = ______
c. m∠ ATD = ______	$m(\widehat{TD})$ = ______
d. m∠ ATE = ______	$m(\widehat{TE})$ = ______
e. m∠ ATF = ______	$m(\widehat{TF})$ = ______
f. m∠ ATG = ______	$m(\widehat{TG})$ = ______

Experiment #3: Chord and Chord Angles





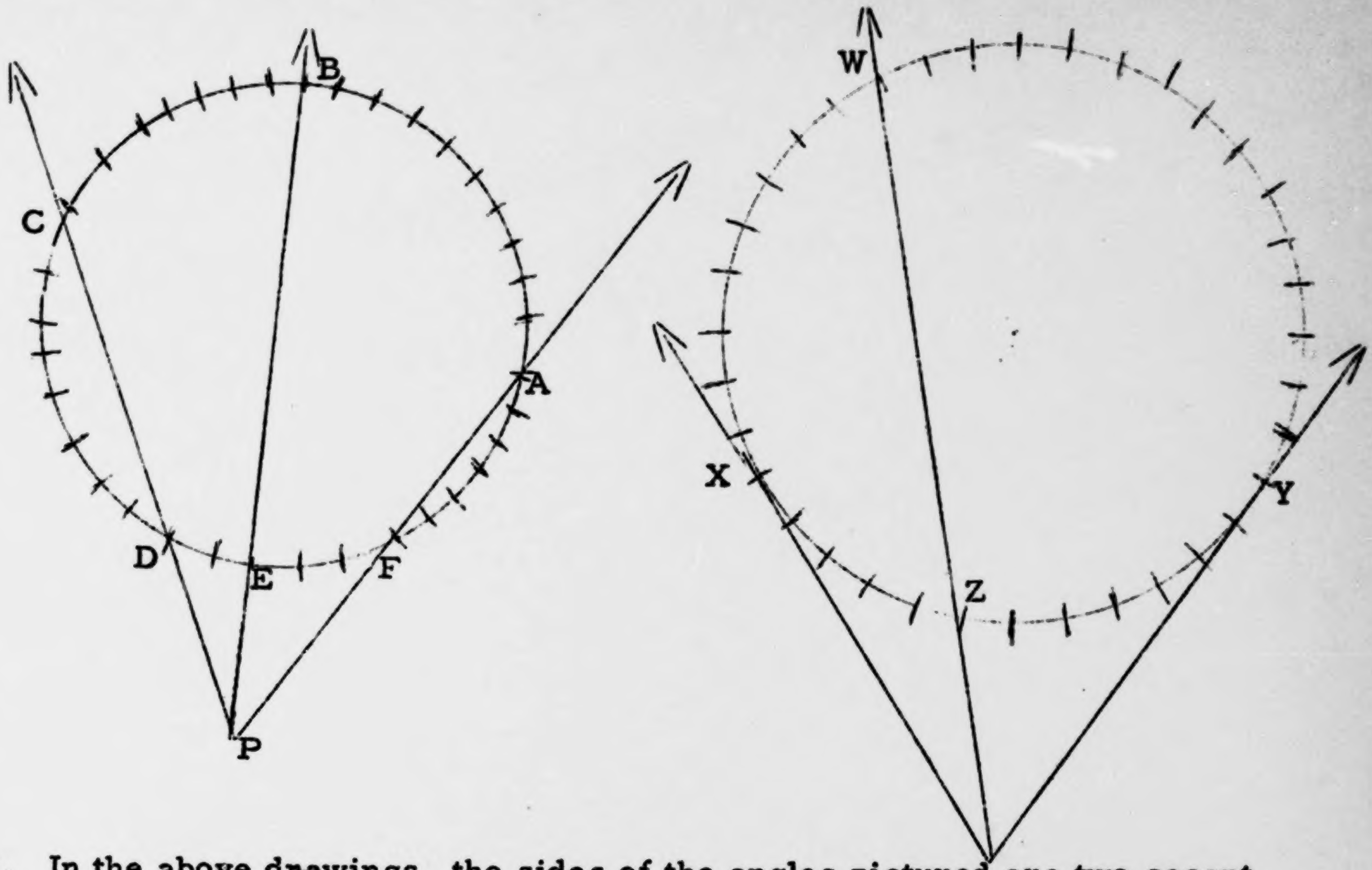
1. In the above drawings, the angles pictured are formed by two chords intersecting at a point in the interior of the circle. With your protractor, measure each of the given angles to the nearest five degrees. Record the measurements in the chart at the bottom of the page.

2. Find the measure of each of the intercepted arcs by counting the number of degrees in each of them. (Each division of the circle represented an arc with a degree measure of ten.) Record these measurements in the chart.

3. Try to discover a method of determining the measure of a chord-chord angle if the measures of the intercepted arcs are given.

Measure of Angle	Measure of Arcs
a. m∠ BPA = ______	$m(\overset{\frown}{AB})$ = ______; $m(\overset{\frown}{DC})$ = ______
b. m∠ APD = ______	$m(\overset{\frown}{AD})$ = ______; $m(\overset{\frown}{BC})$ = ______
c. m∠ YRX = ______	$m(\overset{\frown}{XY})$ = ______; $m(\overset{\frown}{WZ})$ = ______
d. m∠ YRZ = ______	$M(\overset{\frown}{WX})$ = ______; $m(\overset{\frown}{YZ})$ = ______

Experiment #4: Secant-Secant, Tangent-Tangent, and Secant-Tangent Angles



1. In the above ~~drawings~~, the sides of the angles pictured are two secant rays, two tangent rays, or a tangent ray and a secant ray. With your protractor, measure each of these angles to the nearest five degrees. Record the measurements in the chart at the bottom of the page.

2. Find the measure of each of the intercepted arcs by counting the number of degrees in each of them. (Each division of the circle represents an arc with a degree measure of ten.) Record these measurements in the chart.

3. Try to discover a method of determining the measure of a secant-secant angle, a tangent-tangent angle, and a tangent-secant angle if the measures of the intercepted arcs are given.

Measure of Angle	Measure of Arcs
a. $m\angle APB$ = ________	$m(\overset{\frown}{AB})$ = ________ ; $m(\overset{\frown}{EF})$ = ________
b. $m\angle BPC$ = ________	$m(\overset{\frown}{BC})$ = ________ ; $m(\overset{\frown}{DE})$ = ________
c. $m\angle APC$ = ________	$m(\overset{\frown}{AC})$ = ________ ; $m(\overset{\frown}{DF})$ = ________
d. $m\angle YRW$ = ________	$m(\overset{\frown}{YW})$ = ________ ; $m(\overset{\frown}{YZ})$ = ________
e. $m\angle WRX$ = ________	$m(\overset{\frown}{WX})$ = ________ ; $m(\overset{\frown}{XZ})$ = ________
f. $m\angle YRX$ = ________	$m(\overset{\frown}{YWX})$= ________ ; $m(\overset{\frown}{XZY})$ = ________

BIBLIOGRAPHY

Teaching Materials and Aids

A. Books

TP Henderson, Kenneth B., Pingry, Robert E., and Robinson, George A. *Modern Geometry: Its Structure and Function.* New York: McGraw-Hill Book Company, Inc. 1962

TP Jurgensen, Ray C., Donnelly, Alfred J., and Dolciani, Mary P. *Modern Geometry: Structure and Method.* New York: Houghton Mifflin Company. 1963

T Kemeny, John G., Snell, J. Laurie, and Thompson, Gerald L. *Introduction to Finite Mathematics.* Englewood Cliffs, New Jersey: Prentice-Hall. 1957

TP Morgan, Frank M. and Zartman, Jane. *Geometry: Plane · Solid · Coordinate.* New York: Houghton Mifflin Company. 1963

TP The National Council of Teachers of Mathematics. *Multi-Sensory Aids in the Teaching of Mathematics.* Eighteenth Yearbook. New York: Bureau of Publications, Teachers College, Columbia University. 1945

T Richardson, Moses. *Fundamentals of Mathematics.* New York: The Macmillan Company. 1958

TP School Mathematics Study Group. *Mathematics for High School: Geometry with Coordinates.* Leland Stanford Junior University. 1963

B. Pamphlets

TP Brant, Vincent and Keedy, Mervin L. *Elementary Logic for Secondary Schools.* New York: Holt, Rinehart and Winston, Inc. 1962

TP Johnson, Donovan A. *Logic and Reasoning in Mathematics.* Atlanta: Webster Publishing Company. 1963

TP Commission on Mathematics. *Appendices.* Princeton, New Jersey: College Entrance Examination Board. 1959

C. Unpublished Material

TP Coffman, Janet. Logic. Baltimore County Public Schools. 1959 (Mimeographed)

D. Filmstrips

High School Geometry Series - Set I (set of filmstrips, silent with text. $45.00 set, McGraw-Hill Book Co., 330 West 42nd Street, New York 36, New York)

- Coordinate Geometry
- Nature of Proof
- Parallel and Perpendicular Lines and Planes
- Properties of Triangles
- Sets of Quadrilaterals
- Similarity

High School Mathematics Club (set of filmstrips, silent with text. $33.00 set, Film Strip of-the-Month Clubs, 355 Lexington Avenue, New York 17, New York)

- Arc and Angle Measurement
- Congruent Triangles
- Geometric Proof Using Coordinates
- Introduction to Co-ordinate Geometry
- Parallelograms and their Properties
- Similar Triangles, Experiment and Deduction
- The Pythagorean Theorem
- The Slope of a Line

CURRICULUM GUIDE B

GEOMETRY

A Tentative Guide

Prepared for Use with the Text

Modern Geometry - Structure and Method

by

Jurgensen, Donnelly, and Dolciani

CONTENTS

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BASIC CONSIDERATIONS AND SPECIFIC SUGGESTIONS

The purpose of this syllabus is to help you to understand and to teach geometry from a modern point of view. It should help you to improve your teaching by

assisting you in the interpretation of the basic text

serving as the core around which you can plan your daily lessons

providing you with teaching techniques which are the outgrowth of successful teaching experiences in classroom situations similar to your own.

Modern geometry is distinguished from traditional geometry by two characteristics:

the fusion of plane, solid, and coordinate geometry into a single course

the use of a few basic concepts to unify and clarify subject matter.

These two characteristics provided the basis for planning the sequence of topics and the content of the course.

Organization and Use of the Guide

In a mathematical system there are certain basic assumptions. For the effective use of this geometry guide there are certain basic considerations which, if agreed upon at the outset, will make this syllabus a useful tool for the teacher. The committee expects the teacher to keep in mind these considerations throughout the use of this syllabus.

The Use of the Syllabus

The teacher will use and follow the syllabus. While the text has been followed to a considerable degree, there are a number of important additions and deviations.

Effective use of this guide will enable students to see that geometry is an abstract deductive system and a valuable model for explaining and relating ideas about the real world.

The Use of the Teacher's Manual

The teacher will utilize the teacher's manual provided in the basic text. The comments in the teacher's manual are valuable and have not been duplicated in the guide. Additional comments will be found in the parenthetical remarks in the syllabus. These comments present related teaching suggestions that will reinforce the learning of a concept.

The Use of References

The page references listed in the syllabus have been kept to a minimum and have been included only when they are helpful.

The Use of Coordinate Methods

Since the larger percentage of Baltimore County students will have completed Algebra II, coordinate methods in geometry are easily introduced. By utilizing synthetic and coordinate methods in proof, students will become more flexible in their approach to a problem. Flexibility will be important for future work in mathematics.

The Format of Deductive Proof

All students should learn to present good deductive proofs. Efficient use of students' proofs on the board should insure that even the slowest student acquires a knowledge of proof.

The Use of the Text

A judicious interpretation of the basic text is essential for the effective implementation of the outline. The content of the basic text used in this syllabus reflects the recommendations of the Commission on Mathematics of the College Entrance Examination Board and the School Mathematics Study Group.

Only one of the concepts in this text requires clarification. The term "half-plane" as first described on page 31 gives no indication as to whether the half-plane contains the edge. However, on page 35 of the text it is evident that the description is that of the closed half-plane; that is, one where the edge is included in the set of points called the half-plane. This description is neccessary in order to include the straight angle and its measure in the geometry. However, in all subsequent references to the half-plane, except those which specifically refer to the straight angle, the inference made is that of the open half-plane; that is, one in which the edge does not belong. If the teacher keeps this in mind, there should be no difficulty.

Students should learn to depend upon their reading of the text rather than expect the teacher to explain all concepts.

Specific Suggestions

The Use of Student Notebooks

Each student should be required to keep a notebook which would include lists of important definitions, postulates, and theorems. These lists will be most useful to the student if he classifies them under headings relating to methods of proof. Some suggested headings are:

Methods of Proving Segments Equal
Methods of Proving Angles Equal
Methods of Proving Angles Supplementary
Methos of Proving Triangles Congruent
Methos of Proving Lines Perpendicular
Methods of Proving a Triangle is a Right Triangle
Methods of Proving a Triangle is Isosceles
Methods of Proving Lines are Parallel

The Use of Drills

A short drill at the beginning of the period should be a part of each lesson. It should be placed in a designated area so that students can begin work on it as soon as they come into the classroom. Each drill should be designated to serve at least one of three pruposes:

To lead to the intuitive development of a concept which is to be developed formally in the lesson

To provide necessary practice of important concepts over an extended period of time. This often results in more effective learning than that achieved by overconcentration on a topic for a limited period of time.

To review basic concepts of algebra which are needed in the development of this approach to geometry. It is suggested that drills be used in teaching much of the material on real numbers. Teachers should include numerical examples of these properties early in the year rather than wait for the topic to appear in the syllabus.

The Use of Time

While a time allotment has been included for each unit, it is important for the teacher to gauge time carefully by making an overview of the unit and utilizing block planning. Where the time allotment indicates that several days will be spent on a section, extra days can frequently be devoted to additional exercises and board work. Teachers are urged to read the problems of each set of written exercises and their solutions prior to giving assignments. Many of the exercises in the C group should be included, especially those on the space geometry topics. Under normal circumstances Units One to Seven should be completed in the first semester.

Unit	Name	Number of Days	Cumulative Total
One	Elements of Geometry	11	11
Two	Induction: A Method of Discovery	8	19
Three	Deduction and Proof	12	31
Four	Angle Relationships;Perpendicular Lines	11	42
Five	Parallel Lines and Planes	10	52
Six	Congruent Triangles	16	68
Seven	Similar Polygons	22	90
	End of First Semester		
Eight	Circles, Arcs, and Angles	15	105
Nine	Methods of Coordinate Geometry	15	120
Ten	Proofs Using Coordinate Geometry	11	131
Eleven	Areas of Polygons and Circles	15	146
Twelve	Areas and Volumes of Solids	15	161
Thirteen	Constructions and Loci	18	179
		Total for Year	179

MATERIALS OF THE COURSE IN GEOMETRY

Modern Geometry: Structure and Method by Jurgensen, Donnelly and Dolciani will be used as the basic text.

The references to the right of the scope in the outline indicate helpful or additional materials for the teacher and pupils.

Code		Name of Reference
JDD	-	Jurgensen, Donnelly, and Dolciani. *Modern Geometry: Structure and Method*
TM	-	________ *Modern Geometry: Structure and Method* - "Teacher's Manual" (included in Teacher's Edition of the above text.)
HPR	-	Henderson, Pingry, and Robinson. *Modern Geometry: Its Structure and Function*
MZ	-	Morgan and Zartman. *Geometry: Plane · Solid · Coordinate*
SMSG	-	School Mathematics Study Group. *Mathematics for High School: Geometry with Coordinates*
C	-	Coffman. *Logic*
BK	-	Brant and Keedy. *Elementary Logic for Secondary Schools*
J	-	Johnson. *Logic and Reasoning in Mathematics*

	CODE	PAGES
UNIT ONE		
ELEMENTS OF GEOMETRY		
(Time allotment - 11 days)		
I. Introduction		
A. Historical background	MZ....	1-2
1. "Geo-metrein" - earth measure 2. Rope stretchers		
B. Modern needs for geometry		
1. Practical applications 2. Logical reasoning 3. Creative thinking 4. Appreciation of environment 5. Enjoyment	MZ....	2-3
II. Representation of sets (1-1)	JDD....	1-5
(The topics concerning sets should be review for the students. The time required for these topics will depend upon their background. Under no circumstances should these topics be expanded to a complete treatment of sets.)		
A. Concept of set		

SCOPE		REFERENCES	
		CODE	PAGES
B. Basic terms			
member or element ($\in$) non-member ($\notin$) rule method roster method	finite set infinite set the empty or null set (ϕ) equal sets		
III. Use of sets			
A. Types of subsets (1-2)			
1. Subset 2. Proper subset 3. Improper subset			
B. Operations on sets (1-2)			
1. Intersection ($A \cap B$) 2. Union ($A \cup B$) 3. Complement ($\overline{A}$)			
C. Venn diagrams (1-3)			
1. Universal set 2. Diagrams for intersection and union			
IV. Real numbers and the number line (1-4), (1-5)		JDD	11-17
A. Basic terms			
rational real number irrational real number real number line	coordinate graph one-to-one correspondence		

SCOPE	REFERENCES CODE	PAGES
B. Characteristics of the real number line		
1. Every point on the line can be paired with exactly one real number. 2. Every real number can be paired with exactly one point on the line.		
C. Inequalities and symbols		
(The amount of drill necessary on inequalities will depend on the background of the student.)		
V. Distance between points: absolute value (1-5)	JDD....	18-21
A. Informal development: The distance between two distinct points A and B (denoted as AB) on a number line is a unique positive real number.		
(The teacher should develop the concept intuitively by counting the number of units between the two points on the number line.)		
B. Formal development: absolute value		
1. The distance between two points on the number line is expressed by the absolute value of the difference of their coordinates. 2. $\|x\| = x$ where x is a positive number or zero $(x \geq 0)$ $\|x\| = -x$ where x is a negative number $(x < 0)$		
(It is incorrect to define the absolute value of a number as "the number without its sign.")		
VI. Undefined terms (1-7)	JDD....	22-26

SCOPE	REFERENCES	
	CODE	PAGES
A. The necessity and importance of undefined terms		
(The teacher should show that any attempt to define every term will eventually lead to a circular definition.)	HPR...	123
B. Basic undefined terms of geometry		
1. Point(s)		
2. Line (A line is assumed to be a straight line.)		
3. Plane		
C. Intersection of two geometric figures		
D. Drawings to represent intersection		
1. Vocabulary of description		
2. Cardboard models, etc.		
VII. Definitions (1-8)	JDD....	26-30
A. The importance and necessity of good definitions		
1. To delimit the area of discussion		
2. To avoid circularity		
B. Characteristics of a good definition		
1. The term must be stated.		
2. The term must be expressed as a subset of a set.		

3. The definition must indicate how the term is different from all other members of the set.
4. The definition must not contain any unnecessary properties.
5. The definition must be reversible.

C. Description versus definition

(Not all descriptions are definitions, but all definitions are descriptions. Listing properties does not make a definition.)

D. Definitions and symbols

space
collinear points
coplanar points
betweenness of points
line segment ($\overline{AB}$)
length of segment (AB)

equal segments
midpoint of segment
bisector of segment
ray ($\overrightarrow{AB}$)
endpoint of ray
opposite rays

VIII. Angle (1-9)

JDD....30-32

A. Definition of angle

(The definition of angle implies two distinct rays.)

B. Parts of an angle

1. Sides of an angle
2. Vertex of an angle

SCOPE	REFERENCES	
	CODE	PAGES
C. Notation for an angle		
1. Single letter ($\angle A$, $\angle a$) 2. Three letters ($\angle BAC$) 3. A numeral ($\angle 1$)		
D. Related terms		
1. Half-plane		
(The teacher should again refer to the section in the introduction about the half-plane.)		
2. Edge of a half-plane		
(In this text the edge belongs to the half-plane in order that there will be a straight angle.)		
3. Interior of an angle		
(A straight angle has no interior.)		
4. Exterior of an angle		
IX. Measurement of angles (1-10)	JDD....	33-38
A. Use of the protractor		
(The protractor is treated as an undefined tool.)		

SCOPE	REFERENCES CODE	PAGES

1. Proper placement
2. Other protractors
    a. Radian
    b. Quadrant
    c. Mil

SMSG...151-155

B. Basic assumption: There is a one-to-one correspondence between rays in the half-plane and the real numbers from 0 to 180 inclusive.

C. Definitions of related terms

1. Measure of an angle
2. Degrees
3. Minutes
4. Seconds

X. Some special angles and angle relationships (1-11)

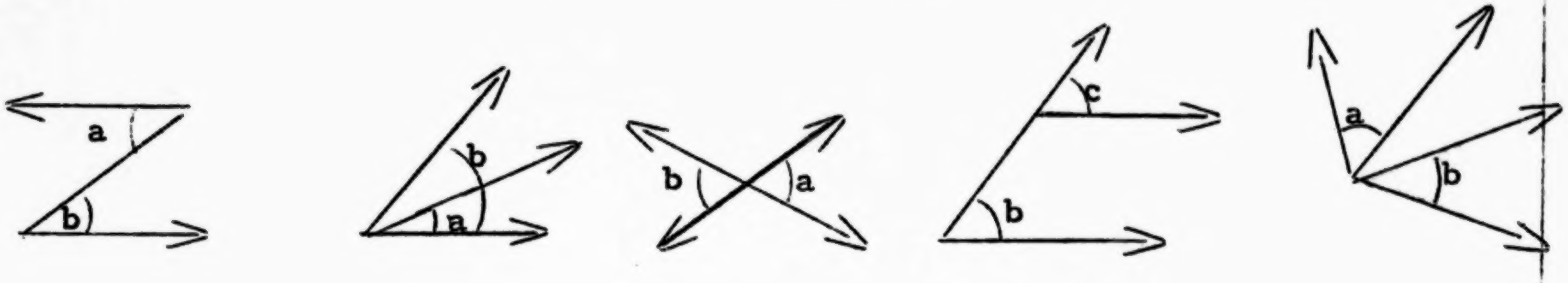
JDD....39-43

A. Definitions

acute angle
right angle
obtuse
straight angle
adjacent angles
exterior sides of adjacent angles
bisect an angle

vertical angles
complementary angles
supplementary angles
equal angles
dihedral angle
face of a dihedral angle
bisector

(In this text ":" is used to mean "is" or "are" to complete a good definition.)

SCOPE	REFERENCES	
	CODE	PAGES
(The following are not adjacent angles.)  	SMSG...	181
B. Use of the protractor 1. To copy an angle 2. To bisect an angle		
XI. Review	JDD....	43-51
A. Review of foregoing material		
B. Enrichment for all classes: "Father of Greek Mathematics"		
C. Enrichment for capable classes: "Extra for Experts"		
XII. Test		

SCOPE	REFERENCES	
	CODE	PAGES
UNIT TWO		
INDUCTION: A METHOD OF DISCOVERY		
(Time allotment - 8 days)		
(The teacher should stress the inductive approach, since the objective of the unit is to develop a knowledge of the inductive process while using new vocabulary.)		
I. Meaning of induction and intuition (2-1)		
A. Introduction: Inductive thinking is a valuable method of discovering probable conclusions.	**JDD...**	**53-57**
B. Definitions	**TM...**	**9**
1. Induction		
2. Intuition		
(The accompanying oral exercises are excellent for class discussion.)		
II. Pairs of lines		
A. Intersecting lines	**JDD...**	**22**
B. Perpendicular lines	**JDD...**	**59**

SCOPE	REFERENCES	
	CODE	PAGES
C. Parallel lines, segments, and rays	JDD...	65
D. Skew lines	JDD...	153
III. Triangles (2-2)	JDD...	32 B-D
(Most of this section is developed in the filmstrip Properties of Triangles by McGraw-Hill.)	JDD...	57-61
A. Definition of triangle		
1. Parts		
a. Vertices b. Sides c. Angles		
2. Separation: A triangle separates a plane into three subsets.		
a. The triangle itself		
b. The interior of the triangle: A point is in the interior of a triangle if and only if it is in the interior of each angle of the triangle.		
c. The exterior of the triangle: A point is in the exterior of a triangle if and only if it is not in the interior or on the triangle.		

3. Exterior angle of a triangle: An exterior angle is the angle formed by one side of the triangle and the adjacent side extended through the common vertex.

(The teacher should emphasize that we often speak of "exterior angles, one at each vertex.")

4. Remote interior angles: nonadjacent interior angles

B. Nomenclature

(The relationship of triangles should be illustrated by the use of set diagrams.)

1. Triangles classified by sides

    a. Scalene triangle
    b. Isosceles triangle

        legs
        base
        vertex
        vertex
        base angles

    c. Equilateral triangle

2. Triangles classified by angles

    a. Acute triangle
    b. Obtuse triangle

SCOPE	REFERENCES	
	CODE	PAGES
c. Right triangle		
right angle hypotenuse legs		
d. Equiangular triangle		
3. Lines related to triangles		
a. Perpendicular lines, segments, or rays b. Perpenducular bisector of a segment		
(A perpendicular bisector may be a line, segment, ray, or plane.)		
c. Altitude d. Median		
IV. General polygons (2-3)	JDD...	61-64
A. Definition of polygon		
B. Related terms		
Vertices Sides Angles		
Convex polygon Consecutive vertices Consecutive sides Diagonal		

SCOPE | REFERENCES CODE | PAGES

C. Nomenclature

1. Polygons classified by number sides

triangle - 3 sides
quadrilateral - 4 sides
pentagon - 5 sides
hexagon - 6 sides
septagon - 7 sides
octagon - 8 sides
nonagon - 9 sides
decagon - 10 sides
dodecagon - 12 sides
n-gon - n sides

2. Special polygons

a. Equilateral
b. Equiangular
c. Regular

V. Quadrilaterals (2-4) — JDD...65-67

A. Definition of a quadrilateral

B. Nomenclature

(The relationships of the quadrilaterals should be illustrated by the use of set diagrams.)

1. Trapezoid

a. Bases
b. Legs
c. Isosceles trapezoid
d. Median

SCOPE	REFERENCES	
	CODE	PAGES
2. Parallelogram		
a. Rhombus		
b. Rectangle		
c. Square		
C. Skew quadrilateral		
VI. Circles (2-5)	JDD...	67-70
(No more than two days should be spent on Section VI, VII, VIII, and IX.)		
A. Definition of circle		
1. Center		
2. Separation: A circle separates a place into 3 subsets.		
a. The circle itself		
b. The interior of the circle		
c. The exterior of the circle		
B. Lines and segments pertaining to circles		
1. Radius		
2. Chord		
3. Diameter		
4. Secant		
5. Tangent		
a. Point of tangency - point of contact		
b. Length of the tangent		

SCOPE

VII. Sets of two or more circles (2-6)

A. Two circles

1. Concentric circles
2. Two circles which intersect in two points: common chord
3. Two tangent circles

   a. Externally tangent circles
   b. Internally tangent circles

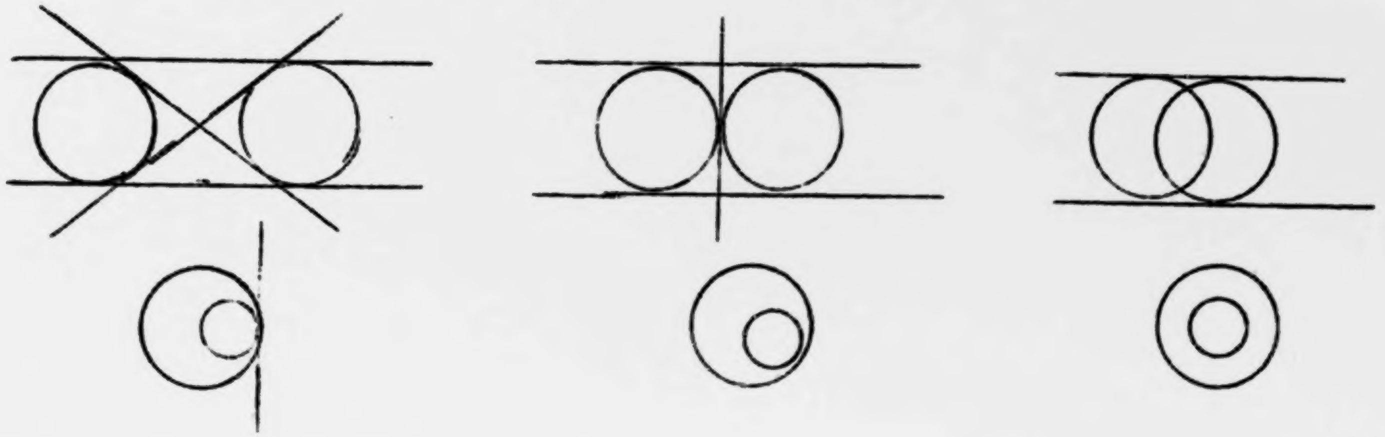
B. Common tangent

1. Kinds of tangents

   a. Internal tangent
   b. External tangent

2. Number of tangents

(The following diagrams should suggest the possibilities:)



REFERENCES	
CODE	PAGES
JDD...	70-73

SCOPE	REFERENCES	
	CODE	PAGES
VIII. Circles and polygons		
A. Inscribed polygon and circumscribed circle		
B. Inscribed circle and circumscribed polygon		
IX. Spheres (2-7)	JDD...	74-76
A. Definition of sphere		
B. Vocabulary		
C. Separation of space by a sphere		
D. Tangent plane		
X. Review		
A. Review of inductive reasoning	JDD...	76-82
B. Enrichment for all classes: review of quadratics (These problems provide excellent drill material.)	JDD...	84-85
XI. Test		

SCOPE	REFERENCES CODE	REFERENCES PAGES
UNIT THREE		
DEDUCTION AND PROOF		
(Time allotment - 12 days)		
I. Deductive reasoning (3-1)	JDD	87-91
A. Early development of deductive reasoning		
1. Hindus 2. Greeks	HPR	123
B. The nature of deductive reasoning		
1. The syllogism		
a. General statement: major premise b. Particular statement: minor premise c. Conclusion		
2. The solution of nongeometrical syllogisms		
(The teacher should use Euler classification circles.)		
C. Deduction versus induction		
1. Advantages of induction		
a. Experimentation is a step in discovery. b. Analogy often suggests important results.		

SCOPE	REFERENCES	
	CODE	PAGES
2. Limitations of induction		
a. Optical illusions are misleading. b. A counterexample invalidates a conclusion reached by inductive reasoning.	MZ....	51-59
3. Advantages of deduction 4. Limitations of deduction: Deductions are based upon assumed statements which may or may not be true.		
II. "If-then" statements (3-2)	JDD...	91-93
A. "If-then" form	TM...	14
1. Identification of the hypothesis 2. Identification of the conclusion 3. Translation of a simple statement into "if-then" form 4. Conditional statement: implication		
a. $p \longrightarrow q$: is read "p implies q" or "if p, then q" b. Antecedent (p) c. Consequent (q)		
(The teacher with a capable class may wish to refer to the teacher's manual p. 14. The following truth table should also be considered only for the capable student.)		

B. Truth table for implication

C. 1-30
BK 1-95
J. 1-59

p	p	$p \longrightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

(In geometry only rows one and two of the truth table will be of value.)

C. Informal development of equivalent statements

1. An implication and its contrapositive
2. The converse and inverse of an implication

(At this time the vocabulary is not important. The student should develop the general notion of what he may or may not consider equivalent statements by use of the exercises.)

D. Algebra problems in "if-then" form (3-3) JDD. . . 94-95

III. Definitions and postulates in geometry (3-4) JDD. . . 95-98

A. Essentials of a mathematical system

1. Undefined terms (review)

SCOPE	REFERENCES CODE	 PAGES
2. Definitions (review)		
a. Examples of faulty definitions		
b. "If and only if" form		
("An angle is a right angle if and only if it has a measure of 90" means the following: 1. If an angle is a right angle, then it has a measure of 90, and 2. If an angle has a measure of 90, then it is a right angle.)		
3. Assumptions		
4. Theorems		
5. Logic: deductive reasoning		
B. Postulates		
1. Statements accepted as true		
2. Equivalent terms		
a. Assumptions		
b. Axioms		
c. Properties		
IV. Properties of real numbers, equality, and inequality	JDD...	98-103
(Division by zero is not defined) (At this point the student should not be expected to exhibit mastery in the use of these properties. Students should realize that algebra is a mathematical system and that each step of a proof should be supported by an appropriate reason.)		

SCOPE	REFERENCES	
	CODE	PAGES
A. Operation of addition 1. Closure property 2. Associative property 3. Commutative property 4. Additive identity 5. Additive inverse B. Operation of multiplication 1. Closure property 2. Associative property 3. Commutative property 4. Multiplicative identity 5. Multiplicative inverse C. Distributive property of multiplication over addition D. Additional properties 1. Order property 2. Zero product-factor property E. Definitions 1. Subtraction 2. Division		

SCOPE	REFERENCES	
	CODE	PAGES
F. Properties of equality		
1. Reflexive property		
2. Symmetric property		
3. Transitive property		
4. Substitution principle		
5. Addition property		
6. Subtraction property		
7. Multiplication property		
8. Division property		
G. Properties of inequality		
1. Transitive property		
2. Addition property		
3. Subtraction property		
4. Multiplication property		
5. Division property		
V. Points, lines, and planes (3-6)	JDD...	104-108
A. Postulates		
1. A line contains at least two points; a plane contains at least three points not all on one line; and space contains at least four points not all in one plane. (P_1)		
2. There is exactly one line through two different points. (P_2)		

SCOPE

REFERENCES
CODE | PAGES

3. There is exactly one plane through any three points which are not on one line. (P_3)

4. If two points lie in a plane, then the line containing them lies in that plane. (P_4)

5. If two different planes intersect, then their intersection is a line. (P_5)

(The teacher should emphasize that the postulates assume relationships between the undefined terms.)

B. Definition of a theorem

(Mathematicians try to use as few postulates as possible. The first theorems are consequences of postulates and definitions.)

C. Theorems

1. If two lines intersect, they intersect in exactly one point. (Thm. 1)
2. If a point lies outside a line, exactly one plane contains the line and the point. (Thm. 2)

3. If two lines intersect, exactly one plane contains both lines. (Thm. 3)

(Students should not be expected to present written supporting arguments; however, they should be able to present reasonably well-organized arguments orally. No distinction should be made about indirect proof at this time.)

D. Exactly one, one and only one, and determine

SCOPE	REFERENCES	
	CODE	PAGES
VI. Lines and segments (3-7)	JDD...	108-112
A. Postulates		
1. Between any two points there is a unique distance. (P_6)		
(This postulate formalizes the intuitive results of sections I-VI. Postulate 6 essentially is the statement that the end result of measurement however performed is a number and that in the background somewhere there must have been some unit in terms of which the measurement was made.)		
2. Ruler Postulate: The set of points on a line can be put in one-to-one correspondence with the real numbers in such a way that:		
a. Any point can be paired with zero;		
b. The distance between any two points is equal to the absolute value of the difference between the numbers corresponding to those points. (P_7)		
(This postulate establishes the right to put a coordinate system on any line, and explains how to find the distance between two points on it.)		
B. Theorems		
1. If three different points are on a line, at most one is between the other two. (Thm. 4)		

SCOPE	REFERENCES	
	CODE	PAGES
2. On a ray there is exactly one point at a given distance from the end point of the ray. (Thm. 5)		
3. A segment has exactly one midpoint. (Thm. 6)		
VII. Two-column deductive proof (3-8)	JDD...	112-114
A. Meaning of proof		
1. Review of syllogism 2. Supporting reasons		
a. Definitions b. Postulates c. Previously proved theorems d. Properties of real numbers.		
B. Form of proof		
1. Given: hypothesis 2. To prove: conclusion 3. Figure 4. Two-column form		
a. Statements b. Reasons		
(The class should study sample proofs.)		

SCOPE	REFERENCES	
	CODE	PAGES
VIII. Review	JDD...	116-120
A. Nature of deductive proof		
B. Enrichment for all classes: "Standards of Length"	JDD...	115
C. Enrichment for capable classes: "Extra for Experts"	JDD...	120-121
IX. Test		

SCOPE

REFERENCES

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UNIT FOUR

ANGLE RELATIONSHIPS; PERPENDICULAR LINES

(Time allotment - 11 days)

I. Angle postulates and betweenness (4-1)

JDD...123-130

A. Review of symbols involving angles

1. Meaning of ∠ABC = ∠RST
2. Meaning of ∠ABC + ∠DEF = ∠XYZ

B. Postulates

1. Angle measurement postulate: To every angle there corresponds a unique real number greater than 0 and less than or equal to 180.

2. Protractor postulate: In a half-plane the set of rays with a common endpoint in the edge of the half-plane can be put in one-to-one correspondence with the real numbers from 0 to and including 180 in such a way that:

(a) Either ray in the edge of the half-plane can be paired with 0;

(b) Any angle whose sides are rays of the given set has a measure equal to the absolute value of the difference between the numbers corresponding to its sides.

(The above two postulates formalize the intuitive concepts which were developed in section IX of Unit I. These postulates are the counterparts of the two postulates on segments.)

SCOPE	REFERENCES	
	CODE	PAGES
C. Betweenness of rays: Ray OB is said to lie between rays OA and OC when:	MZ	10
1. All three rays have a common endpoint;		
2. All three rays lie in the union of a half-plane and its edge; (The teacher should refer again to the introduction on the half-plane.)		
3. The real numbers x, y, z, which correspond to rays OA, OB, and OC, respectively, are such that either $z > y > x$ or $x < y < z$.		
II. Angle relationships (4-1)	JDD	125-130
A. Theorems		
1. Angle-addition theorem: If ray OE lies between rays OD and OF in a half-plane, then $\angle DOE + \angle EOF = \angle DOF$. (Thm. 7)		
2. In a half-plane, through the endpoint of a ray lying in the edge of the half-plane, there is exactly one other ray such that the angle formed by the two rays has a given measure. (Thm. 8)		
3. An angle has exactly one bisector. (Thm. 9)		
B. Use of angle theorems		
1. When a diagram is given, it is permissible to assume a betweenness relation of points or rays.		

SCOPE	REFERENCES	
	CODE	PAGES
2. When a diagram is given, it is not permissible to assume a measure relation of angles or segments.		
(It is important for the teacher to work an example using inequalities with the class.)		
III. Proofs involving straight angles, right angles, and perpendicular lines	JDD...	130-137
A. Review of definitions		
straight angle right angle perpendicular lines complementary angles supplementary angles adjacent angles		
B. Theorems		
1. All straight angles are equal. (Thm. 10)		
2. If the exterior sides of two adjacent angles are opposite rays, the angles are supplementary. (Thm. 11)		
(The term"linear pair"might be used.)	SMSG..	179-180
3. All right angles are equal. (Thm. 12)		
4. If two lines are perpendicular, they meet so as to form right angles. (Thm. 13)		
5. If two lines meet so as to form a right angle, the lines are perpendicular. (Thm. 14)		

SCOPE	REFERENCES	
	CODE	PAGES
6. If two adjacent acute angles have their exterior sides in perpendicular lines, the angles are complementary. (Thm. 15)		
7. In a plane, through a point in a line, there is exactly one perpendicular to the line. (Thm. 16)		
(The phrase "in a plane" should be carefully discussed.)		
IV. Proofs of theorems in two-column form (4-3)	JDD...	137-143
A. Supporting arguments in two-column form		
B. Illustrative proofs		
1. If two angles are supplementary to the same angle or to equal angles, they are equal to each other. (Thm. 17)		
2. If two angles are complementary to the same angle or to equal angles, they are equal. (Thm. 18)		
3. If two lines intersect, the vertical angles formed are equal. (Thm. 19)		
V. The demonstration of a theorem (4-4)	JDD...	143-145
(Students should be given the opportunity to criticize proofs. This can be accomplished by having the students write their proofs on the board.)		

SCOPE	REFERENCES	
	CODE	PAGES
A. Six essential parts		
1. Statement 2. Figure 3. Given 4. To prove 5. Analysis 6. Proof		
B. Helpful suggestions		
1. The figure should depict the given conditions. 2. The statement should be rephrased in "if-then" form. 3. The plan of proof may be discovered by reasoning from the conclusion to the given facts. 4. Only given data, definitions, postulates, previously proved theorems, and algebraic properties are acceptable as reasons. 5. Previously proved theorems may suggest methods of attack.		
VI. Review		
A. Methods of proving two angles equal	JDD...	145-148
B. Enrichment for all classes: "A Twentieth Century Mathematician"	JDD...	149
C. Drill for all classes: Cumulative Test, Chapters 1-4	JDD...	150-151
VII. Test		

SCOPE	REFERENCES CODE PAGES
UNIT FIVE **PARALLEL LINES AND PLANES** **(Time allotment - 10 days)**	
I. Introduction	**JDD...32 A-F**
A. Drawing lines and planes in space	**SMSG..335**
(The teacher might begin an informal introduction of this unit by having each student draw a box. Examination of the student drawings should illustrate the need for agreement on a uniform method of representation.)	
B. Relationships of lines and planes in space	
(A list of these relationships can be found in SMSG, pp. 599-601. The use of the overhead projector and models is helpful. The student should be required to make a number of drawings of some of the possible situations; however, this is not meant to be a thorough discussion.)	**SMSG..599-601**

SCOPE	REFERENCES	
	CODE	PAGES
II. Basic properties (5-1)		
A. Definitions	JDD	150-159
(The filmstrip Parallel and Perpendicular Lines and Planes, McGraw-Hill, is excellent as an introduction to these definitions.)		
parallel lines skew lines a line parallel to a plane parallel planes line perpendicular to a plane perpendicular planes horizontal lines vertical lines edge of a dihedral angle plane angle of a dihedral angle measure of a dihedral angle dihedral angles classified by their plane angles adjacent dihedral angles vertical dihedral angles complementary dihedral angles supplementary dihedral angles	MZ	78-80
B. Theorem: If two parallel planes are cut by a third plane, the lines of intersection are parallel. (Thm. 20)		
C. Intuitive relationships for space	MZ	166-170
(These topics could be stated as formal definitions or proved as theorems, but time does not permit it. The teacher should be satisfied when the student is cognizant of these concepts through illustrations and exercises. The exercises on pp. 155-156 in the basic text are especially good. The concepts developed in these exercises are not listed elsewhere.)		

SCOPE	REFERENCES CODE PAGES
III. Transversals and special angles (5-2)	JDD...157-163

A. Definition of transversal

B. Special angles

interior angles
exterior angles
corresponding angles
alternate interior angles
alternate exterior angles
consecutive interior angles

C. Postulate: If two parallel lines are cut by a transversal, the corresponding angles are equal. (P_{10})

D. Theorems

1. If a transversal of two parallel lines is perpendicular to one of them, it is perpendicular to the other one also. (Thm. 21)

2. If two parallel lines are cut by a transversal, alternate interior angles are equal. (Thm. 22)

3. If two parallel lines are cut by a transversal, the sum of the measures of a pair of consecutive interior angles is 180. (Thm. 22A, Problem 18, p. 162)

(The teacher may wish to spend several days on this section, depending upon the facility with which the class has grasped formal proof.)

<table>
<tr><th rowspan="2">SCOPE</th><th colspan="2">REFERENCES</th></tr>
<tr><th>CODE</th><th>PAGES</th></tr>
<tr><td>IV. Indirect proof (5-3)</td><td>JDD...</td><td>163-166</td></tr>
<tr><td>A. The negative of a statement

(If the students ask for the negative of a compound sentence, the teacher should refer to TM p. 23.)

<table><tr><th>p</th><th>$\sim p$</th></tr><tr><td>T</td><td>F</td></tr><tr><td>F</td><td>T</td></tr></table></td><td></td><td></td></tr>
<tr><td>B. Examples of indirect proof

1. Nongeometrical situations
2. Geometrical situations</td><td></td><td></td></tr>
<tr><td>C. Structure of indirect proof

1. List of all possibilities
2. Supposition: the negative of the proposed conclusion
3. Contradiction of a known fact
4. Paragraph form

(Both direct and indirect proofs are often written in paragraph form. References should be made to some of the supporting arguments which are good proofs in paragraph form.)</td><td>SMSG..</td><td>374-375</td></tr>
<tr><td>V. The parallel postulate and its implications (5-4)</td><td>JDD...</td><td>166-170</td></tr>
</table>

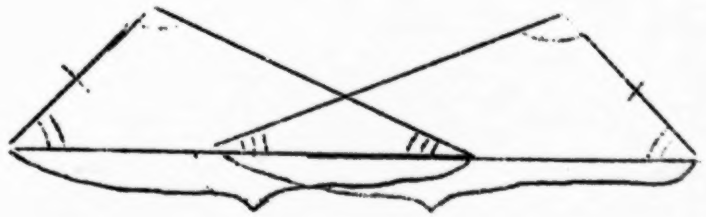
SCOPE	REFERENCES	
	CODE	PAGES
A. Postulate: Through a point outside of a line, there is exactly one parallel to that line. (P_{11})		
B. Inductive development of a converse	TM	23-24
1. Writing the converse 2. Examination of the truth of a converse 3. The contrapositive 4. The inverse		
(The statement is equivalent to the contrapositive, not the converse. Some teachers may wish to introduce a truth table as a way of demonstrating logical equivalence.)	C	16-17
C. Theorem: Through a point outside a line, exactly one perpendicular can be drawn to the line. (Thm. 23)		
VI. Converse of earlier statements about parallels (5-15)	JDD	171-175
A. Use of an auxiliary line		
B. Theorems		
1. If two lines are cut by a transversal so that the corresponding angles are equal, the lines are parallel. (Thm. 24)		
2. In a plane, if two lines are perpendicular to a third line, they are parallel to each other. (Thm. 25)		
3. If two lines are cut by a transversal so that alternate interior angles are equal, the lines are parallel. (Thm. 26)		

SCOPE	REFERENCES	
	CODE	PAGES
VII. Applying parallels to triangles (5-6)	JDD	176-180
A. Theorem: The sum of the measures of the angles in a triangle is 180.(Thm. 27)		
B. Corollaries: definition		
1. If two angles of one triangle are equal to two angles of another triangle, the third angles are equal also.		
2. Each angle of an equilateral triangle has a measure of 60.		
3. A triangle can have at most one right angle or one obtuse angle.		
4. The acute angles of a right triangle are complementary.		
5. The sum of the measures of the angles of a quadrilateral is 360.		
C. Theorem: The measure of an exterior angle of a triangle is equal to the sum of the measures of the two remote interior angles. (Thm. 28)		
VIII. Review		
A. Summary of types of proofs	JDD	181-186
1. Direct: indirect		
2. Paragraph: two-column		
B. Enrichment for all classes		
1. Practical applications		
2. "Non-Euclidean Geometers"	JDD JDD	180 187
C. Enrichment for capable students: "Extra for Experts"	MZ	142
IX. Test	JDD	186

SCOPE	REFERENCES	
	CODE	PAGES
UNIT SIX		
CONGRUENT TRIANGLES		
(Time allotment - 16 days)		
I, Informal treatment of triangles	JDD...	189-193
A. Properties of the triangle		
1. Rigidity of the triangle		
2. Determination of the triangle		
B. Review of the vocabulary of triangles		
C. A view of congruence		
1. Coincidence		
2. One-to-one correspondence between vertices	MZ....	92
3. Corresponding parts of two triangles	SMSG..	225-230
a. Three pairs of corresponding angles		
b. Three pairs of corresponding sides		
D. Marking corresponding parts		
1. Included angle		
2. Included side		

SCOPE	REFERENCES	
	CODE	PAGES
(The teacher should judge by the ability of the students how fast to proceed with formal proof of congruent triangles. This is the time to concentrate on writing and explaining proofs at the board. For a very slow class the teacher may wish to introduce one postulate each day, selecting the appropriate problems using that postulate. At this point the student should always state the postulates and theorems completely in his proofs.)		
II. Formal treatment of congruent triangles (6-2)		
(An introduction to this section can be found in Congruent Triangles, a Filmstrip of the Month.)	JDD...	193-197
A. Congruence: formal definition		
B. Properties of congruence		
1. Reflexive		
2. Symmetric		
3. Transitive		
C. Postulates		
(The teacher should use experiments to introduce the postulates.)	SMSG.. HPR...	249-253 46-50
1. If three sides of one triangle are equal to three sides of another triangle, the triangles are congruent. (SSS) (P_{12})		

SCOPE	REFERENCES CODE	REFERENCES PAGES
2. If two sides and the included angle of one triangle are equal to two sides and the included angle of another triangle. the triangles are congruent. (SAS) (P_{13})		
3. If the hypotenuse and a leg of one right triangle are equal to the hypotenuse and a leg of another right triangle, the triangles are congruent. (HL) (P_{14})		
4. If two angles and the included side of one triangle are equal to two angles and the included side of another triangle, the triangles are congruent. (ASA) (P_{15})		
III. More ways to prove triangles congruent (6-3)	JDD...198-201	
(The following theorems and corollaries should be de-emphasized, since they are special cases of the postulates.)		
A. Theorem: If the legs of one right triangle are equal to the legs of another right triangle, the triangles are congruent. (LL) (Thm. 29)		
B. Theorem: If two angles and any side of one triangle are equal to the corresponding parts of another triangle, the triangles are congruent. (AAS) (Thm. 30)		
1. Corollary: If the hypotenuse and an acute angle of one right triangle are equal to the hypotenuse and an acute angle of another right triangle, the triangles are congruent. (HA)		

SCOPE	REFERENCES	
	CODE	PAGES
2. Corollary: If a leg and an acute angle of one right triangle are equal to the corresponding parts of another right triangle, the triangles are congruent. (LA) (The teacher should emphasize that when using postulates and theorems concerning the congruence of right triangles, the student must first establish that the triangle is a right triangle.)		
IV. Overlapping triangles (6-4)	JDD...	201-203
A. Drawings of overlapping triangles (Many students have difficulty with proofs involving overlapping triangles. The teacher should make use of the overhead projector and of drawings with colored chalk and cross-hatching or shading in the presentation of overlapping triangles. Requiring students to outline the triangles in different colors is helpful in planning the proof.) (Marking corresponding parts may be done in this way:) 		

SCOPE	REFERENCES	
	CODE	PAGES
B. Information given by a figure		
V. Proving corresponding parts equal by the definition of congruent triangles: CPCTE. (6-5)	JDD...	204-207
VI. Isosceles triangles (6-6)	JDD...	207-212
A. Conditions placed upon auxiliary lines	MZ....	110-111
1. Overdetermined 2. Underdetermined 3. Determined		
B. Theorem: If two sides of a triangle are equal, then the angles opposite those sides are equal. (Base angles of an isosceles triangle are equal.) (Thm. 31)		
1. Corollary: An equilateral triangle is also equiangular. 2. Corollary: Each angle of an equilateral triangle has a measure of 60.		
C. Theorem: If two angles of a triangle are equal, then the sides opposite those angles are equal. (Thm. 32)		
Corollary: An equiangular triangle is also equilateral.		
(Some of the exercises involving congruent triangles in space should be included.)		
(Although inequalities are not an integral part of the development of geometry in the basic text, the student	TM....	21

SCOPE	REFERENCES	
	CODE	PAGES
should be exposed to the intuitive notions of inequality in triangles. If the student experiments by measuring triangles with a ruler and a protractor he should discover:		
1. The sum of two sides of a triangle is greater than the third side.	MZ	490-496
2. An exterior angle of a triangle is greater than either remote interior angle.		
3. If two angles of a triangle are unequal, then the sides opposite these angles are unequal in the same order.		
4. If two sides of a triangle are unequal, then the angles opposite these sides are unequal in the same order.		
5. If two triangles have two sides of one equal to two sides of the other, but the third side of the first triangle is greater than the third side of the second, then the angle opposite the third side of the first is greater than the angle opposite the third side of the second.		
6. If two triangles have two sides of one equal to two sides of the other, but the included angle of the first triangle is greater than the included angle of the second, then the third side of the first is greater than the third side of the second.)		

SCOPE	REFERENCES CODE	 PAGES
VII. Applying properties of congruent triangles to quadrilaterals (6-7)	JDD...	213-217
A. Review of definitions for quadrilaterals		
B. Definitions		
1. The distance from a point to a line (or a plane) 2. The distance between two parallel lines (or planes) 3. The distance from a point to a figure		
C. Properties of quadrilaterals		
(Since the properties of quadrilaterals are developed in exercises 1-18, 21 and 22 on pp. 215-216 in the basic text, much care should be taken in their proof. The class should realize that the results are indeed theorems to be used as reasons in later proofs. The teacher may wish to use the following filmstrips: Parallelograms and Their Properties, Film Strip of-the-Month; Sets of Quadrilaterals, McGraw-Hill.)		
VIII. Practical applications	JDD...	217-218
IX. Review: Enrichment for all classes	JDD...	220-227
A. "The Height of Forgetfulness"	JDD...	221
B. "Extra for Experts"	JDD...	226-227
X. Test		

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	CODE	PAGES
UNIT SEVEN		
SIMILAR POLYGONS		
(Time allotment - 22 days)		
I. Ratio and proportion (7-1)	JDD...	229-233
A. Ratio		
1. The ratio of a to b is the number $\frac{a}{b}$ (a:b)		
2. The notation a:b:c is an abbreviation for the three ratios, a:b, b:c, and a:c.		
B. Proportion		
1. A proportion is a statement that two ratios are equal.		
2. Terms of a proportion		
a. First, second, third, and fourth		
b. Means and extremes		
c. Proportional sequences of numbers		
II. Special properties of a proportion (7-2)		
A. Means-extremes product property (multiplication property)	JDD...	234-237

SCOPE	REFERENCES	
	CODE	PAGES
B. Equivalent-form property	MZ....	264-265
1. Alternation 2. Inversion		
C. Denominator-addition property (addition property)		
D. Denominator-subtraction property		
E. Numerator-denominator sum property of equal ratios		
III. Similarity (7-3)	JDD...	235-243
A. Informal discussion of similarity		
B. Formal definition of similar polygons: Two polygons are similar if and only if they have:		
1. A one-to-one correspondence between vertices; 2. Corresponding angles equal; 3. Corresponding sides in proportion.		
C. Theorem: If two polygons are similar, the ratio of their perimeters equals the ratio of any pair of corresponding sides. (Thm. 33)		
IV. Similar triangles (7-4)	JDD...	244-250
A. Properties of similarity for triangles		
1. Reflexive 2. Symmetric 3. Transitive		

SCOPE

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CODE PAGES

(The filmstrip *Similar Triangles, Experiment and Deduction*, Film Strip of-the-Month, is an excellent introduction to the following section.)

B. Postulates

1. If two angles of one triangle are equal to two angles of another triangle, the triangles are similar. (AA) (P_{16})

   a. Corollary: If two right triangles have an acute angle of one triangle equal to an acute angle of the other, the triangles are similar.

   b. Corollary: If two isosceles triangles have their vertex angles equal, the triangles are similar.

   c. Corollary: If two triangles are similar to a third triangle, the two triangles are similar to each other. (transitive property)

2. Two triangles are similar if an angle of one equals an angle of the other, and the sides including these angles are in proportion. (P_{16A})

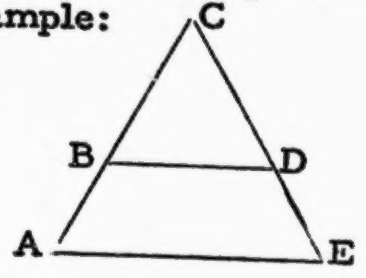
3. Two triangles are similar if their corresponding sides are in proportion (P_{16B})

MZ...297-299

(The above two postulates do not appear in the text. They can be proved but for our purposes are best postulated.)

C. Writing proportions and products from similar triangles

(It is usually difficult for the slower student to establish proportions from similar triangles. While it is important for all students to use the correspondence to help them, certain students will need an additional device. Such a device might be incorporated in the body of the proof with very little difficulty. For example:



Given: $\overline{BD} \parallel \overline{AE}$

Prove: $\frac{AC}{BC} = \frac{EC}{DC}$

Statements:

1. $\overline{BD} \parallel \overline{AE}$

In $\triangle ACE$ and $\triangle BCD$

2. $CE \mid \angle CAE = \angle CBD \mid CD$
3. $CA \mid \angle CEA = \angle CDB \mid CB$
4. $AE \mid \angle ACE = \angle BCD \mid BD$
5. $\therefore \triangle ACE \quad \triangle BCD$

6. $\frac{CE}{CD} = \frac{CA}{CB} = \frac{AE}{BD}$

The proof would begin in a normal fashion, except for the listing of the triangles which are involved. It is necessary to list all three pairs of equal angles. When similarity has

SCOPE

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been established, the student should go back and list the side opposite each angle of the appropriate triangle. The proportion is then readily established.)

V. Properties of special segments (7-5)

JDD...251-258

A. Definitions

1. Segments divided proportionally
2. Sides of a triangle divided proportionally

B. Theorems

1. If a line is parallel to one side of a triangle and intersects the other two sides, it divides them proportionally. (Thm. 34)

   a. Corollary: If three or more parallel lines intersect two transversals, they divide them into proportional segments

   b. Corollary: If three or more parallel planes intersect two lines, they divide them into proportional segments.

2. If a ray bisects one angle of a triangle, it divides the opposite side into segments which are proportional to the two sides adjacent to those segments. (Thm. 35)

3. If two triangles are similar, their corresponding altitudes have the same ratio as any pair of corresponding sides. (Thm. 36)

SCOPE	REFERENCES	
	CODE	PAGES
VI. Properties of the altitude drawn to the hypotenuse in a right triangle (8-6)	MZ....	276
	JDD...	258-264
A. Definitions 1. Mean proportional 2. Projection of a point on a line 3. Projection of a segment on a line in the same plane		
B. Theorem: If the altitude is drawn to the hypotenuse of a right triangle, the two triangles formed are similar to the given triangle and to each other. (Thm. 37)		
1. Corollary: A leg of a right triangle is the mean proportional between the hypotenuse and the projection of that leg on the hypotenuse.		
2. Corollary: The altitude drawn to the hypotenuse of a right triangle is the mean proportional between the segments of the hypotenuse.		
3. Corollary: In a right triangle the product of the hypotenuse and the altitude drawn to it is equal to the product of the legs. (The teacher may wish to use the review material on radicals in the basic text p. 261 as drill for this lesson.)		
VII. The Pythagorean Theorem (7-7)	JDD...	266-269
(The filmstrip, The Pythagorean Theorem, Film Strip of-the-Month, gives a historical introduction to this theorem, proof of the theorem, and applications.)	JDD...	272 G-L

SCOPE	REFERENCES	
	CODE	PAGES
A. Historical background		
B. Theorem: In any right triangle the square of the hypotenuse is equal to the sum of the squares of the legs. (Thm. 38)		
C. The converse: If the sum of the squares of two sides of a triangle is equal to the square of the third side, the triangle is a right triangle. (Thm. 39)		
(The teacher may wish to have the students present contrasting proofs of the Pythagorean Theorem: for example, Euclid's proof and President Garfield's proof.)	MZ	429-431
VIII. Special right triangles: 30-60-90 and 45-45-90 (7-8)	JDD	269-273
(The 30-60-90 triangles and the 45-45-90 triangles are widely used in the solution of practical problems in many related fields, such as industrial arts. The ratios of the measures of the sides of these two triangles are basic to the development of many important concepts in trigonometry. It is essential that students, especially those who continue work in mathematics, acquire a working knowledge of these ratios. It is suggested (1) that these ratios be derived from the Pythagorean Theorem, (2) that students receive immediate practice in their use through many numerical examples and problems dealing with practical applications and the triangles, and (3) that examples of a similar nature be included in daily drills over an extended period of time.		

SCOPE	REFERENCES	
	CODE	PAGES
This procedure should enable students to become thoroughly familiar with these ratios and their use. Since radicals are used in the solution of these problems, the teacher may wish to include problems with radicals in daily drills for several days prior to the teaching of this section. Better students should be able to derive and make use of the following generalization: $\frac{1}{\sqrt{n}} = \frac{\sqrt{n}}{n}$.)		
A. The 30-60-90 relationship		
B. The 45-45-90 relationship		
IX. Three-dimensional figures (7-9)	JDD...	274-278
A. Representation of the right angle in three dimensions	SMSG..	335-341
(The teacher should provide the student with practice in making his own diagrams.)		
B. Basic terms	SMSG..	774-788
polyhedron face edge vertex polyhedral angle trihedral angle rectangular solid diagonal of a rectangular solid regular square pyramid altitude of a pyramid slant height of a pyramid lateral edge of a pyramid	MZ....	126-127

SCOPE	REFERENCES CODE	 PAGES
C. The Pythagorean Theorem in three dimensions		
X. Projections into a plane (7-10) (When models are projected from above the stage of the overhead projector, the effect of projections into a plane is easily illustrated.)	JDD...	278-281
XI. Practical applications	JDD...	281-283
XII. Review	JDD...	283-289
A. Cumulative review	JDD...	310-319
B. Enrichment for all students: "A Pupil of Thales"	JDD...	291
C. Enrichment for capable students: "Extra for Experts"	JDD...	289-290
XIII. Test		

SCOPE	REFERENCES CODE PAGES

UNIT EIGHT

CIRCLES, ARCS, AND ANGLES

(Time allotment - 15 days)

I. Arcs and central angles (9-1)

JDD... 321-327

A. Definitions

- circle
- equal circles
- central angle
- minor arc
- major arc
- semicircle
- measure of a minor arc
- measure of a major arc
- measure of a semicircle
- equal arcs

B. Properties inherent in the definition of a circle

1. Radii of a circle are equal.
2. Radii of equal circles are equal.

C. Arc-addition postulate: If the intersection of $\widehat{DE}$ and $\widehat{EF}$ of a circle is the single point E, then $\widehat{DE} + \widehat{EF} = \widehat{DEF}$. ($P_{17}$)

D. Theorems

1. If two central angles are equal, their arcs are equal. (Thm. 40)
2. If two minor arcs are equal, their central angles are equal. (Thm. 41)

SCOPE	REFERENCES CODE	PAGES
II. Inscribed angles and limits (9-2)	JDD...	327-335

(The filmstrip Arc and Angle Measurement, Film Strip of-the-Month, gives an intuitive introduction to the next two sections.)

A. Definitions

1. Inscribed angle
2. Arc which an angle intercepts
3. Arc in which an angle is inscribed

B. Theorem: The measure of an inscribed angle is one-half of the measure of its intercepted arc. (Thm. 42)

1. Corollary: An angle inscribed in a semicircle is a right angle.

2. Corollary: If a quadrilateral is inscribed in a circle, the opposite angles are supplementary.

3. Corollary: If two inscribed angles intercept the same arc or equal arcs, the angles are equal.

C. Limits

1. Intuitive discussion
2. Description of a limit
3. The use of limits

   a. Theorem: The measure of an angle formed by a secant ray and

SCOPE	REFERENCES	
	CODE	PAGES
a tangent ray drawn from a point on a circle is one-half the measure of the intercepted arc. (Thm. 43)		
b. Corollary: A diameter drawn to the point of contact is perpendicular to the tangent at that point.		
(Several of the exercises in group C on page 335 of the basic text will be essential for later proof of constructions.)		
III. Other angles formed by secants and tangents (9-3)	JDD...	336-340
A. Theorems		
1. The measure of the angle formed by two secants intersecting in the interior of a circle equals one-half the sum of the measures of the intercepted arcs. (Thm. 44)		
2. The measure of the angle formed by two secants intersecting in the exterior of a circle equals one-half of the difference of the measures of the intercepted arcs. (Thm. 45)		
3. The measure of the angle formed by a tangent and a secant, or by two tangents, intersecting in the exterior of a circle equals one-half the difference of the measures of the intercepted arcs. (Thm. 46)		
B. Proofs without the use of limits	JDD...	556-558

SCOPE	REFERENCES
	CODE PAGES
IV. Chords of the same circle or of equal circles (9-4)	JDD...340-344
A. Definitions	
1. To bisect an arc 2. Midpoint of an arc	
B. Theorems	
1. A diameter perpendicular to a chord bisects the chord and its two arcs. (Thm. 47)	
2. In the same circle or equal circles, equal chords have equal arcs. (Thm. 47A)	
3. In the same circle or equal circles, equal arcs have equal chords. (Thm. 47B)	
(The students should prove exercises 11-20 on pages 343-344 in the basic text in order to use the statements as theorems in future work.)	
V. Proportions involving chords, secants, and tangents (9-5)	JDD...345-349
(The teacher should review the meaning of the terms secant segment, and tangent segment as found on page 68 of the basic text.)	

SCOPE	REFERENCES	
	CODE	PAGES
A. Theorems		
1. If two chords intersect in the interior of a circle, the product of the segments of one chord equals the product of the segment of the other. (Thm. 48)		
2. If two secants are drawn to a circle from an exterior point, the product of one secant and its external segment is equal to the product of the other secant and its external segment. (Thm. 49)		
3. If a tangent and a secant are drawn to a circle from an exterior point, the tangent is the mean proportional between the secant and its external segment. (Thm. 50)		
B. Numerical examples		
VI. Practical applications	JDD...	349-350
VII. Review	JDD...	350-355
A. Enrichment for all classes: "A Child Prodigy"	JDD...	357
B. Enrichment for capable classes: "Extra for Experts"	JDD...	355-356
VIII. Test		

UNIT NINE

METHODS OF COORDINATE GEOMETRY

(Time allotment - 15 days)

One of the goals of modern geometry is the development of analytic geometry simultaneously with synthetic geometry. It should be emphasized that both are deductive systems and that it is useful to have more than one method of approach to a proof.

The teacher should proceed through the earlier sections of this unit on coordinate methods as rapidly as the students' ability and algebra background will permit. This will make it possible for all students to use coordinate methods in simple proofs and the most capable students to use coordinate methods in more difficult proofs. Unit Nine is a tool unit which has been included to provide background for the study of coordinate proofs in Unit Ten. The use of coordinates adds flexibility to geometry by providing an additional method of proof. The ability to use coordinates in proof is the goal of these two coordinate geometry units.

I. Graphs on the number line (11-1) — JDD...393-396

A. Definitions

locus
graph
set-builder notation
axis
inclusive
exclusive

JDD...376

(A set of points is a locus, whether the points are designed by a graph or the set-builder notation.)

SCOPE	REFERENCES	
	CODE	PAGES
B. Definition of one-dimensional coordinate system: A one-to-one correspondence between the set of points on a line and the set of real numbers is called a one-dimensional coordinate system.		
C. The graphs of solution sets of equalities and inequalities		
II. Plotting points in two-dimensions (11-2)	JDD...	396-399
A. Definitions		
ordered pair x-axis y-axis origin quadrant coordinate axes x-coordinate: abscissa y-coordinate: ordinate		
B. Definition of two-dimensional coordinate system: A one-to-one correspondence between the set of points in the plane and the set of ordered pairs of real numbers is called a two-dimensional coordinate system.		
(A one-to-one correspondence between the set of points in space and the set of ordered triples of real numbers is called a three-dimensional coordinate system. The teacher should include exercises which make use of three-dimensional coordinates.)	SMSG..	633-643
C. Symmetry (11- 3)	JDD...	399-402
1. Symmetry with respect to a point 2. Symmetry with respect to a line 3. Symmetry with respect to a plane	MZ....	200-202

SCOPE	REFERENCES	
	CODE	PAGES
III. Graphs meeting given conditions (11-4)	JDD...	402-404
IV. The distance formula (11-5)	JDD...	404-406
(The filmstrip Introduction to Coordinate Geometry, Film Strip of-the-Month, gives a review of basic concepts and develops the distance formula.)		
A. Informal development		
B. Theorem: The distance between two points (x_1, y_1) and (x_2, y_2) is		
$d = (x_2 - x_1)^2 + (y_2 - y_1)^2$. (Thm. 51)		
(The three-dimensional distance formula should be included. The text treats this informally in exercises 19-22 on page 406.)	JDD...	272-J
V. The circle (11-6)	JDD...	407-409
A. Equation of a circle with center at the origin and radius r: $x^2 + y^2 = r^2$.		
B. Theorem: The circle with center (a, b) and radius r has the equation		
$(x - a)^2 + (y - b)^2 = r^2$. (Thm. 52)		
C. The interior of the circle: $(x - a)^2 + (y - b)^2 < r^2$		
D. The exterior of the circle: $(x - a)^2 + (y - b)^2 > r^2$		

SCOPE	REFERENCES CODE	 PAGES
VI. The midpoint formula (11-7)	JDD...	410-412
A. Inductive development		
B. Theorem: The midpoint of the segment joining the points (x_1, y_1) and (x_2, y_2) is the point $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$. (Thm. 53)		
VII. Slope		
(The filmstrip The Slope of a Line, Film Strip of-the-Month Club, introduces the concept of slope and considers the parallel line relationship inductively.)		
A. Intuitive concept of slope		
B. The slope of a segment		
C. The slope of a line		
VIII. Parallel and perpendicular lines (11-9)	JDD...	415-418
A. Two nonvertical lines are perpendicular if and only if they have equal slopes. (Thm. 54)		
B. Two nonvertical lines are perpendicular if and only if the slope of one line is the negative reciprocal of the slope of the other line. (Thm. 55)		
(The teacher should emphasize the significance of "if and only if" in theorems.)		

SCOPE	REFERENCES CODE | PAGES
IX. Writing equations of lines (11-10)	JDD...419-423
A. Theorems	
1. The graph of any equation which can be written in the form $ax + by = c$, a and b not both 0, is a line. (Thm. 56)	
2. The equation of the line passing through the point (x_1, y_1) and having the slope m is $y - y_1 = m(x - x_1)$. (Thm. 58) (Point-slope form)	
Corollary: The equation of the line containing points (x_1, y_1) and (x_2, y_2) is $y - y_1 = m(x - x_1)$, where $m = \frac{y_2 - y_1}{x_2 - x_1}$ and $x_2 \neq x_1$.	
B. Graphs and equations	
X. Graphing of lines (11-11)	JDD...424-427
A. The use of intercepts to determine a line	
B. The use of a point and a slope to determine a line	
C. The intersection of two lines	
(The teacher should review the graphing of inequalities. The half-plane referred to at the bottom of page 426 in the basic text is a half-plane where the edge is not included.)	MZ....504-505 JDD...427-433

SCOPE	REFERENCES	
	CODE	PAGES
XI. Review (The filmstrip Coordinate Geometry, McGraw-Hill, offers a good review of this unit.)		
A. Enrichment for all classes: "Descartes"	JDD...	434
B. Enrichment for capable classes: "Extra for Experts"	JDD...	435
XII. Test		

SCOPE	REFERENCES CODE | PAGES
UNIT TEN **PROOFS USING COORDINATE GEOMETRY** **(Time allotment - 11 days)**	
I. Placing coordinate axes (12-1)	**JDD...437-442**
(The filmstrip Geometric Proof Using Coordinates, Film Strip of-the-Month Clubs, covers the problem of placing the axes in an informative and delightful manner.)	
A. Placing the axes on the figure	
B. Using the properties of a figure to assign coordinates to points	
II. Parallel and perpendicular lines (12-2)	**JDD...442-444**
(In this section and in the successive ones, the main body of each lesson should consist of sample proofs and the techniques for achieving various results. Many of the exercises proved in this unit are theorems which are not proved elsewhere in the text. The exercises should be selected carefully, and they should include: page 449, exercises 1, 2, 3, 7; page 451, exercises 2, 4, 5, 7; pages 453-454, exercises 1, 4, 5; pages 455-456, exercises 10, 11; and pages 457, exercises 23, 24.	

SCOPE	REFERENCES	
	CODE	PAGES
The teacher should emphasize that the use of coordinate methods presents an alternative approach to the solution of a problem. For any problem the choice must be made between direct proof, indirect proof, or proof by coordinate methods. All are deductive methods.)		
III. Distances (12-3)	JDD...	445-448
A. Proving two segments bisect each other		
B. Finding points of trisection		
IV. Triangles (12-4)	JDD...	448-449
V. Parallelograms (12-5)	JDD...	450-451
VI. Trapezoids and general quadrilaterals (12-6)	JDD...	452-454
VII. Review	JDD...	454-457
A. Cumulative review of Units VIII to X	JDD...	464-469
(It will be necessary to omit constructions and locus exercises at this time.)		
B. Enrichment for all classes: "The Fourth Dimension"	JDD...	458-459
C. Enrichment for capable classes: "Extra for Experts"	JDD...	460-463
VIII. Test		

SCOPE	REFERENCES CODE	 PAGES
UNIT ELEVEN		
AREAS OF POLYGONS AND CIRCLES		
(Time allotment - 15 days)		
I. The meaning of area (13-1)	JDD...	471-474
A. Definitions		
1. Polygonal region 2. Area of a polygonal region 3. Unit of area		
B. Postulates		
1. To every polygonal region there corresponds a unique positive number. (P_{18})		
2. If two triangles are congruent, they have the same area. (P_{19})		
3. Area-addition postulate: The area of every geometric figure is the sum of the areas of the nonoverlapping figures of which it is composed.	HPR...	401
(This postulate replaces postulate 20 in the text. The terms "geometric figure" and "nonoverlapping" are undefined terms. This postulate is more powerful than the one found in the text.)		

SCOPE	REFERENCES	
	CODE	PAGES
II. Areas of rectangles and parallelograms (13-2) A. Postulate: The area of a rectangle is equal to the product of its base and its altitude. (P_{21}) B. Definitions 1. Base of a parallelogram 2. Altitude of a parallelogram C. Theorem: The area of a parallelogram is the product of any base and the corresponding altitude. (Thm. 58) Corollary: Parallelograms having equal bases and equal altitudes are equal in area.	JDD...	474-479
III. Areas of triangles and trapezoids (13-3) A. The area of a triangle is equal to one-half the product of its base and altitude. (Thm. 59) 1. Corollary: Triangles with equal bases and equal altitudes have equal areas. 2. Corollary: The area of an equilateral triangle with side s is one - fourth the product of the side squared and $\sqrt{3}$. $A = \frac{s^2}{4}\sqrt{3}$	JDD...	479-482

SCOPE	REFERENCES	
	CODE	PAGES
3. Corollary: The area of a rhombus is one-half the product of its diagonals. $A = \frac{1}{2}d_1d_2$		
4. Corollary: The area of a triangle with sides a, b, and c is $\sqrt{s(s-a)(s-b)(s-c)}$ where $s = \frac{1}{2}(a + b + c)$		
(Hero's formula should be used by all students but proved only by the most capable.)	JDD...	514-515
B. The area of a trapezoid is equal to one-half the product of its altitude and the sum of its bases. $A = \frac{1}{2}h(b_1 + b_2)$ (Thm. 60)		
IV. Proofs involving areas (13-4)	JDD...	482-484
V. Comparing areas of similar triangles (13-5)	JDD...	484-487
A. Theorem: The ratio of the areas of two similar triangles is the square of the ratio of any two corresponding sides.		
B. Numerical examples		
VI. Special properties of regular polygons (13-6)	JDD...	487-489
A. Theorems		
1. A circle can be circumscribed about any regular polygon. (Thm. 62) 2. A circle can be inscribed in any regular polygon. (Thm. 63)		

SCOPE	REFERENCES	
	CODE	PAGES
B. Definitions 1. Center of a regular polygon 2. Radius of a regular polygon 3. Apothem of a regular polygon 4. Central angle of a regular polygon		
C. Corollary: A central angle of a regular n-gon has a measure of $\frac{360}{n}$.		
VII. Areas of regular polygons (13-7)	JDD...	490-491
A. Theorem: The area of a regular polygon is equal to one-half the product of the apothem and the perimeter. $A = \frac{1}{2} ap$ (Thm. 64)		
B. Numerical examples (The teacher should have the student use 30-60-90 and 45-45-90 relationships in the written exercises.)		
VIII. Comparing areas of similar polygons (13-8)	JDD...	491-493
A. Postulate: If two polygons are similar, they can be separated into the same number of triangles similar each to each and in corresponding positions. (P_{22})		
B. Theorem: The ratio of the areas of two similar polygons is equal to the square of the ratio of a pair of corresponding sides. (Thm. 65)		

SCOPE	REFERENCES	
	CODE	PAGES
IX. The circle as a limiting case of a regular polygon (13-9)	JDD...	494-497
A. As the number of sides of a polygon inscribed in a circle increases:		
1. The apothems increase, approaching the radius of the circle as a limit. 2. The perimeters increase, approaching the circumference of the circle as a limit. 3. The areas increase, approaching the area of the circle as a limit.		
B. Definitions		
1. Circumference of a circle 2. Area of a circle		
C. Theorem: The ratio of the circumference to the diameter is the same for all circles. This constant ratio is called pi (π). (Thm. 66)		
Corollary: The circumference of two circles have the same ratio as their radii and their diameters.		
D. The historical background of pi		
X. Area of a circle (13-10)		
A. Definition of a circular region: A circular region is the union of the circle and its interior.		
B. Theorem: The area of a circle is equal to the product of π and the square of the radius of the circle. $A = \pi r^2$ (Thm. 67)		

SCOPE	REFERENCES	
	CODE	PAGES
C. Corollary: The areas of two circles have the same ratio as the squares of their radii and the squares of their diameters.		
XI. Arcs, sectors, and segments (13-11)	JDD...	500-503
A. Length of an arc: $\frac{\text{length of the arc}}{\text{circumference of circle}} = \frac{\text{degree measure of the arc}}{360}$		
B. Sector of a circle		
1. Definition		
2. Area of a sector : $\frac{\text{area of sector}}{\text{area of circle}} = \frac{\text{degree measure of the arc}}{360}$		
C. Segment of a circle		
1. Definition		
2. Area of a segment: the area of the sector minus the area of the triangle		
XII. Review	JDD...	507-512
Enrichment for all classes: "The Better Half"	JDD...	513
XIII. Test		

UNIT TWELVE

AREAS AND VOLUMES OF SOLIDS

(Time allotment - 15 days)

The content of this unit should be treated informally. Models, pictures and experiments should serve as the basis for building vocabulary and arriving at generalizations.

The student should learn to analyze each problem on the basis of these generalizations. Only basic formulas for areas and volumes should be memorized. Those formulas which can be readily derived from the basic formulas should not be memorized. For example, since a rectangular solid is a special type of prism, the student need only memorize the formula for the volume of a prism.

I. Prisms (14-1, 14-2)

JDD...517-521
MZ....240-242

A. Basic terms

prism
bases
lateral faces
lateral edges
prismatic surface
right prism
oblique prism
parallelepiped
rectangular parallelepiped
cube
altitude of a prism
section of a prism
right section of a prism
lateral area
total area
volume

SCOPE	REFERENCES	
	CODE	PAGES
B. Area	MZ	443
1. Lateral area of a right prism: LA = ph	SMSG	921-924
2. Total area of a right prism: TA = ph + 2B		
C. Volume		
1. Volume of a right prism: V = Bh		
2. Summary of length: area and volume as applied to geometric figure	JDD	528M-P
3. Cavalieri's principle: Given two solids and a plane. If for every plane which intersects the solids and is parallel to the given plane, the two intersections have equal areas, then the two solids have equal volumes.	SMSG	926-927
II. Pyramids (14-1, 14-3)	JDD	521-523
A. Basic terms		
pyramid base vertex lateral faces lateral edges regular pyramid altitude of a pyramid slant height of a regular pyramid lateral area of a pyramid total area of a pyramid volume of a pyramid	SMSG	927-929
B. Area		
1. Lateral area of a pyramid: LA = $\frac{1}{2}$ pl, where l = slant height		
2. Total area of a pyramid: TA = $\frac{1}{2}$ pl + B		

SCOPE	REFERENCES
	CODE PAGES
C. Volume	JDD...528N-P
1. Intuitive derivation of the volume of a pyramid 2. Postulational approach to volume 3. Volume of a pyramid: $V = 1/3\ Bh$	
III. Cylinders (14-4, 14-5)	JDD...524-526
A. Basic terms	
circular cylinder bases lateral surface axis altitude right circular cylinder oblique circular cylinder cylinder of revolution	MZ....359-361
B. Basic concept: Any property of prisms which does not depend on the number of lateral faces holds true for cylinders.	
C. Area	
1. Lateral area of a circular cylinder: $LA = 2\pi rh$ 2. Total area of a circular cylinder: $TA = 2\pi rh + 2\pi r^2$	
D. Volume	
1. Volume of a circular cylinder: $V = \pi r^2 h$ 2. The limit process and volume	

SCOPE	REFERENCES	
	CODE	PAGES
IV. Cones (14-4, 14-6)	**JDD...**	**527-528**
A. Basic terms		
circular cone **vertex** **base** **lateral surface** **axis**		
altitude of a circular cone **right circular cone** **oblique circular cone** **slant height of a right circular cone**		
B. Basic concept: Any property of pyramids which does not depend on the number of lateral faces holds true for cones.		
C. Area		
1. Lateral area of a right circular cone: $LA = \pi r l$		
2. Total area of a right circular cone: $TA = \pi r l + \pi r^2$		
D. Volume of a right circular cone: $V = 1/3\ \pi r^2 h$		
V. Frustums of Pyramids and Cones	**JDD...**	**540-542**
A. Definition: frustum		
B. Lateral area		
(If time is lacking, no numerical exercises should be attempted for this section; however, the material should be discussed.)		

SCOPE	REFERENCES	
	CODE	PAGES
1. Frustum of a regular pyramid: $LA = \frac{1}{2} l (p_1 + p_2)$		
2. Frustum of a right circular cone: $LA = \pi l (r_1 + r_2)$		
C. Volume (optional)		
1. Frustum of a regular pyramid 2. Frustum of a right circular cone		
VI. Spheres (14-7)	MZ	362-363
A. Basic terms	MZ JDD	404 528-530
great circle of a sphere spherical angle spherical polygon spherical triangle lune		
(The teacher should employ the slate globe to illustrate these terms.)		
B. Informal derivations for the sphere	SMSG JDD	923-924 528 Q-R
C. Area of a sphere: $S = 4\pi r^2$		
D. Volume of a sphere: $V = 4/3 \pi r^3$		
VII. Areas of similar solids (14-8)	JDD	530-533
A. Definition of similar solids		
B. Theorem: Lateral area and total area of two similar solids have the same ratio as the squares of any pair of corresponding segments.		

SCOPE	REFERENCES CODE	REFERENCES PAGES
VIII. Volumes of similar solids (14-9)	JDD...	533-535
A. Theorem: The volumes of two similar solids have the same ratio as the cubes of any pair of corresponding segments.		
B. The ratio of volume to total area of similar figures		
IX. Review		
(The teacher should summarize the following concepts:		
1. Any two corresponding segments of similar figures have the same ratio as any two other corresponding segments.		
2. The lateral areas and total areas of similar solids have the same ratio as the squares of any two corresponding segments.		
3. The volumes of two similar solids have the same ratio as the cubes of any two corresponding segments.)		
X. Test		

UNIT THIRTEEN

CONSTRUCTIONS AND LOCI

(Time allotment - 18 days)

The time and detail which should be taken with this unit depend upon the background of the student. The presentations of "proofs" of any constructions may be criticized on the grounds that nothing is really proved. Since constructions are a physical representation of geometrical ideas rather than a mathematical concept, these constructions are not necessary for a logical development of geometry. However, these proofs provide the student with practice in writing proofs of tangible rather than abstract statements and they should be included in the unit. The teacher should stress that the phrase "a set of points" which has been used in the past is actually a description of a locus. Some of the C exercises should always be included.

I. What construction means (10-1) — JDD...359-364

A. Drawing versus construction

B. Rules governing the use of compass and straight edge — MZ...15-17

1. A straight edge can be used only to draw a line.
2. A compass can be used to mark off equal segments and to construct circles or arcs of circles.

(Students will find the use of these standard statements helpful in presenting the steps in constructions: "Draw segment $\overline{AB}$" and "with O as center and $\overline{AB}$ as radius, construct an arc.")

C. Basic assumptions for construction

1. It is possible to construct a line through any two given points.

2. It is possible to construct a circle in a plane with any given point as a center and any given segment as radius.

3. Two nonparallel lines lying in a plane intersect in a point.

4. Two circles intersect in two points if the distance between their centers is less than the sum of their radii but greater than the difference of their radii. The two intersection points will lie on opposite sides of the line of centers.

5. A line and a circle intersect in two points if the line contains a point in the interior of the circle.

D. Constructions

1. Construct, with a given ray as side, an angle equal to a given angle.
2. Construct a ray which bisects a given angle. (midray)

E. The format of a construction problem

1. A statement
2. Given
3. To construct
4. The constructed figure
5. Steps of construction
6. Justification

SCOPE	REFERENCES	
	CODE	PAGES
II. Constructing parallel and perpendicular lines (10-2)	JDD...	364-367
A. In a plane construct a line perpendicular to a given line at a given point on the line.		
B. Construct a perpendicular to a given line from a given point outside the line.		
C. Construct the perpendicular bisector of a given segment.		
D. Construct a line parallel to a given line through a given point not on the line.		
III. Constructions involving circles (10-3)	JDD...	368-372
A. Construct a line which bisects a given arc on a circle.		
B. Construct the tangent to a given circle at a given point of the circle.		
C. Construct the tangents to a given circle from a given external point.		
D. Circumscribe a circle about a given triangle.		
E. Inscribe a circle in a given triangle.		
IV. Constructing special segments (10-4)		
A. Divide a given segment into any given number of equal parts.	JDD...	372-375
B. Construct a segment whose length is the fourth proportional to the lengths of three given segments.		

SCOPE	CODE	PAGES
C. Construct a segment whose length is the mean proportional between the lengths of two given segments.		
D. Construct a figure involving a given area and using a fourth proportional. (13-12)	JDD...	504-505
E. Construct a figure involving a given area and a square root. (13-13)	JDD...	505-506
V. The meaning of locus (10-5)	JDD...	375-379
A. Informal discussion	MZ....	459-465
B. Definition of locus		
C. Essentials of locus solution 1. A diagram in which the figure representing the locus is clearly indicated 2. An accurate word description of the locus		
D. Loci in space	MZ....	476-478
VI. The proof of a locus problem	JDD...	554-556
A. Prove that any point on the locus named satisfies the given condition.		
B. Prove that any point that satisfies the given conditions lies on the locus named. (A locus theorem should be stated in "if and only if" form. Teachers wishing to prove locus theorems with the more capable students will find a discussion of their proofs on pages 554-556 of the basic text.)		

SCOPE	REFERENCES	
	CODE	PAGES
VII. Intersection of loci (10-6)	JDD... MZ ...	379-381 467-473
(The teacher should stress the concurrent lines related to triangles and their relation to loci.)		
VIII. Constructions by means of loci (10-7)	JDD... MZ....	382-384 190-194
IX. Review	JDD...	384-389
A. Enrichment for all classes: "The Three Famous Construction Problems"	JDD...	387
B. Enrichment for capable classes: "Extra for Experts"	JDD...	390-391
X. Test		

BIBLIOGRAPHY

Teaching Materials and Aids

A. Books

TP Henderson, Kenneth B., Pingry, Robert E., and Robinson, George A. *Modern Geometry: Its Structure and Function.* New York: McGraw-Hill Book Company, Inc. 1962

TP Jurgensen, Ray C., Donnelly, Alfred J., and Dolciani, Mary P. *Modern Geometry: Structure and Method*. New York: Houghton Mifflin Company. 1963

T Kemeny, John G., Snell, J. Laurie, and Thompson, Gerald L. *Introduction to Finite Mathematics.* Englewood Cliffs, New Jersey: Prentice-Hall. 1957

TP Morgan, Frank M. and Zartman, Jane. *Geometry: Plane · Solid · Coordinate*. New York: Houghton Mifflin Company. 1963

TP The National Council of Teachers of Mathematics. *Multi-Sensory Aids in the Teaching of Mathematics*. Eighteenth Yearbook. New York: Bureau of Publications, Teachers College, Columbia University. 1945

T Richardson, Moses. *Fundamentals of Mathematics.* New York: The Macmillan Company. 1958

TP School Mathematics Study Group. *Mathematics for High School: Geometry with Coordinates.* Leland Stanford Junior University. 1963

B. Pamphlets

TP Brant, Vincent and Keedy, Mervin L. *Elementary Logic for Secondary Schools.* New York: Holt, Rinehart and Winston, Inc. 1962

TP Johnson, Donovan A. *Logic and Reasoning in Mathematics.* Atlanta: Webster Publishing Company. 1963

TP Commission on Mathematics. *Appendices.* Princeton, New Jersey: College Entrance Examination Board. 1959

C. Unpublished Material

TP Coffman, Janet. Logic. Baltimore County Public Schools. 1959 (Mimeographed)

D. Filmstrips

High School Geometry Series - Set I (set of filmstrips, silent with text. $45.00 set, McGraw-Hill Book Co., Inc., 330 West 42nd Street, New York 36, New York)

- Coordinate Geometry
- Nature of Proof
- Parallel and Perpendicular Lines and Planes
- Properties of Triangles
- Sets of Quadrilaterals
- Similarity

High School Mathematics Club (set of filmstrips, silent with text. $33.00 set, Film Strip of-the-Month Clubs, 355 Lexington Avenue, New York 17, New York)

- Arc and Angle Measurement
- Congruent Triangles
- Geometric Proof Using Coordinates
- Introduction to Co-ordinate Geometry
- Parallelograms and their Properties
- Similar Triangles, Experiment and Deduction
- The Pythagorean Theorem
- The Slope of a Line